THE THEORY OF OPTIMIZING YOUR TELESCOPE TO DETECT THE FAINTEST STARS

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Abstract

The theory of optical imaging of point and extended sources is used to develop formulae describing object-to-sky contrast ratios. Study of these formulae will allow one to discover how to optimize an instrument for a particular observing program.

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The brightness of a point source as perceived by a detector in the focal plane of a telescope or by the eye viewing that image through an eyepiece is proportional to the power delivered to that image by the objective. The flux, F_o, of star light incident upon the objective has units of energy per second, W, per unit area. The total power collected, P, is calculated from equation (1) where A is the area of an objective with diameter, D.

$$P = F_o \cdot A = W \cdot \pi \cdot \frac{D^2}{4} \qquad (1)$$

It comes as no surprise to see that the brightness increases with the square of the objective diameter. All telescope users soon become aware that a given star appears brighter in a larger instrument.

The second case to consider is that of an extended source such as a galaxy, nebula, comet, or patch of the sky itself. Figure 1 shows an image formed by an objective with diameter, D, and focal length, f. A resolution element in the focal plane is described by a square with dimensions α by α. This element can be a pixel of a diode array, photographic grain, or photometer diaphragm. It has angular dimension on one side of α divided by F which is the angle θ measured in radians.

The brightness of the source is best expressed in units of energy per second per unit area per unit solid angle. This is the "specific intensity" measure, I_o, successfully used by radio astronomers to describe extended sources. In order to obtain the power delivered to the resolution element, one multiplies the specific intensity of the source by the area of the objective, A, and the solid angle, Ω, of the resolution element. This solid angle is just the product of θ by θ or $(\alpha/F)^2$ and is in units of steradians. Equation (2) gives the power, S, in the pixel.

$$S = I_o \; A \; \Omega = W_o \cdot \frac{\pi}{4} \cdot D^2 \left(\frac{\alpha}{F}\right)^2 = W_o \cdot \frac{\pi}{4} \; \alpha^2 \cdot \left(\frac{D}{F}\right)^2 \qquad (2)$$

Equation (2) can be rewritten by substituting the focal ratio f/n for the quantity F/D giving equation (3).

$$S = W_o \cdot \frac{\pi}{4} \cdot \alpha^2/(f/n)^2 \qquad (3)$$

Equation (3) is useful with direct photography of extended objects. It tells one that the power in a resolution element can be increased by using a film with coarser grain (increase α) or faster (smaller) f-ratio.

Dividing equation (1) by equation (2) gives the star-to-sky power ratio.

$$\frac{P}{I} = \frac{W \frac{\pi}{4} D^2}{W_o \frac{\pi}{4} D^2 \left(\frac{\alpha}{F}\right)^2} = \frac{W \quad F^2}{W_o \quad \alpha^2} \tag{4}$$

Equation (4) is of special interest to photoelectric observers. Since the best results are obtained with the highest signal-to-noise ratio, the star-to-sky ratio can be increased in two equivalent ways. Either increase the focal length or decrease the diaphragm diameter. The signal-to-background ratio changes with the square of α or F while guiding difficulties are proportional to the first power of these quantities.

It may seem easy to change α since most photometers have interchangeable diaphragms but the focal length of a telescope seems fixed. Do not forget the usefulness of a negative lens near the focal point for changing the effective focal length of a system. Such a lens is the traditional "Barlow lens."

So far direct imaging, not visual viewing, has been considered. The extended object equations (2) and (3) will need modification. Refering to Figure 2, one sees that the real image in the focal plane appears to the eye as a virtual image projected on the sky with dimensions θ' by θ'. The intensity of this virtual image is the power in the focal plane pixel divided by the apparent solid angle, Ω'. Ω' is calculated in equation (5).

$$\Omega' = \theta'^2 = \left(\frac{\alpha}{f}\right)^2 \tag{5}$$

The conversion to intensity is performed in equation (6).

$$I' = \frac{S}{\Omega'} = \left(W_o \frac{\pi}{4} \frac{\alpha^2}{f/n}\,2\right) / \left(\frac{\alpha}{f}\right)^2 = W_o \frac{\pi}{4} \frac{f^2}{(f/n)^2} = W_o \frac{\pi}{4} \frac{f^2}{F^2/D^2} = W_o \frac{\pi}{4} \frac{D^2 f^2}{F^2} \tag{6}$$

Equation (6) can be rewritten in still other useful forms. The magnification, M, is given in equation (7).

$$M = \frac{\theta'}{\theta} = F/f \tag{7}$$

The quantity f/F in equation (6) can be replaced by 1/M to give

$$I' = \frac{W_o \pi}{4} \cdot \frac{D^2}{M^2} \tag{8}$$

The magnification can also be given in terms of the objective diameter, D, and the exit pupil diameter, p_x.

$$M = D/p_x \tag{9}$$

Substituting M from equation (9) into equation (8) gives

$$I' = W_o \frac{\pi}{4} \cdot p_x^2 \tag{10}$$

which indicates that p_x should be maximized for the most intense image. Unfortunately, the quantity p_x cannot be increased without limit. Most sources quote 7 mm as the largest size for the well-dark-adapted eye. To avoid having your eye be the effective aperture stop, p_x should be limited to 5 or 6 mm.

Equations (6), (8), and (10) are equally valid expressions for the image intensity. The quantity p_x is no more important a variable than M, f/n, or F. They are strictly interrelated. One equation may be more convenient than another in a particular case.

Dividing equation (10) evaluated for the object by equation (10) evaluated for the sky gives the signal-to-sky ratio.

$$S/N = \frac{W \frac{\pi}{4} \quad p_x^2}{W_o \frac{\pi}{4} \quad p_x^2} = \frac{W}{W_o} \qquad (11)$$

All instrumental factors disappear, giving only the original object-to-sky contrast. This means that if one is viewing galaxies or comets in a light-polluted area, one cannot supress the sky brightness without having the same thing happen to the object.

All is not futile in this case. A property of the eye is that it can best perceive a very faint diffuse source if the diameter is about 1/2 degree on the retina and the source intensity exceeds the background intensity by about 5 percent (Kinsman 1986). Selection of the appropriate magnification to give a 1/2 degree image is easily done. Obtaining a source-to-background intensity ratio greater than or equal to 1.05 depends on the object chosen and local light pollution. Waiting for the moon to set or traveling to a darker site are the only options left, unless the object is an emission line source such as a planetary nebula. Then, interference filters may be used to supress the sky intensity significantly while passing most of the nebular light.

Fortunately, the case of viewing a star against a bright sky gives a more encouraging result. Dividing equation (1) by equation (8) gives

$$\frac{P}{I'} = \frac{W \frac{\pi}{4} D^2}{W_o \frac{\pi}{4} \frac{D^2}{M^2}} = \frac{W}{W_o} M^2 \qquad (12)$$

This indicates that the contrast ratio can be increased without limit! Such is not really the case since the image of a star is not a true point. The image is at least a diffraction disk plus a variable seeing blur.

Equation (12) is valid as long as the star image is smaller than the resolution element of the eye. Allen (1976) quotes the theoretical resolution of the eye cells as 25 arc sec for the cones and 12 arc sec for the rods. The actual resolution is about 1 arc min. These values are for high-contrast images only.

Figure (3) shows the intensity profiles of three star images blurred by seeing. Each star has the same halfpower halfwidth or undergoes the same atmospheric image spreading. The wings of each image extend for quite a distance from the center. There is a visual threshold beyond which the eye does not detect the image. This threshold defines the apparent diameter of the star. Thus, the faintest star has the smallest apparent image diameter. A still fainter star will not be perceived at all.

If the low-contrast resolution of the eye is 5 arc min (300 arc sec), then a faint star becomes an extended object when the seeing disk exceeds 5 arc min on the retina. Figure (4) shows the image brightness vs. magnification for the sky and three stars of different magnitude. The sky continues to decrease in intensity while the stars remain constant as magnification increases. When the critical magnification for each star in turn is reached, the intensity of the star starts to decrease with a constant ratio to the sky.

Quantitatively, one can estimate the critical magnification for the best contrast with a faint star. If the seeing disk is 3 arc sec for a bright star, the fainter stars will appear as 1 or 1/2 arc sec images. A magnification of 300 to 600 will enlarge these fainter images up to the 300 arc sec resolution of the eye. Magnification beyond this is counter-productive since the intensity of the star image will decrease with the square of the magnification.

Under average seeing conditions, a larger telescope will usually experience worse seeing conditions than a smaller one next to it. This means that the smaller telescope may be able to use higher magnifications and actually attain its theoretical magnitude limit while the larger telescope reaches the extended object made before its theoretical limit can be attained. The larger telescope will show fainter stars but at a rate of diminishing returns because of seeing.

From the equations developed here, the observer (photographic, photoelectric, or visual) can optimize his system with respect to signal-to-background ratio or the apparent intensity. Experienced observers are already aware of most of the principles outlined in this paper. The details of the theoretical calculations are given so that they can have a quantitative basis for their intuition and extend details of the theory to their own particular observing interest.

As far as enhanced coatings or special eyepiece designs are concerned, anything that increases the output of a telescope system will help the power in an image but will not change the image-to-background ratios. Each observer soon finds that he has a favorite eyepiece which will bring out the faint stars when others fail. This eyepiece is not necessarily of superior design but closely matches the magnification to the observer's resolution. If the observer has determined his low-contrast resolution then he can estimate the critical magnification on a given night. A collection of eyepieces with small differences in focal length around the average critical focal length will be useful.

Some parting words on the eye as a detector are in order. The eye should be well-dark-adapted. This is not achieved in just a few hours at the telescope but must be a constant endeavor. Efficient sunglasses should be considered for daily out-of-doors activity. Beware of inefficient but stylish designer models. Sufficient oxygen in the blood also increases the sensitivity of the eye. A few deep breaths before an observation will make the faintest objects more apparent.

Finally, practice is essential. One must learn to recognize the feeble scintillations that indicate the presence of a faint star. Remember the difficulty a visitor to your observatory has in seeing a conspicuous small galaxy for the first time.

REFERENCES

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Kinsman, G. 1986, **Sky & Telescope 72**, 392.

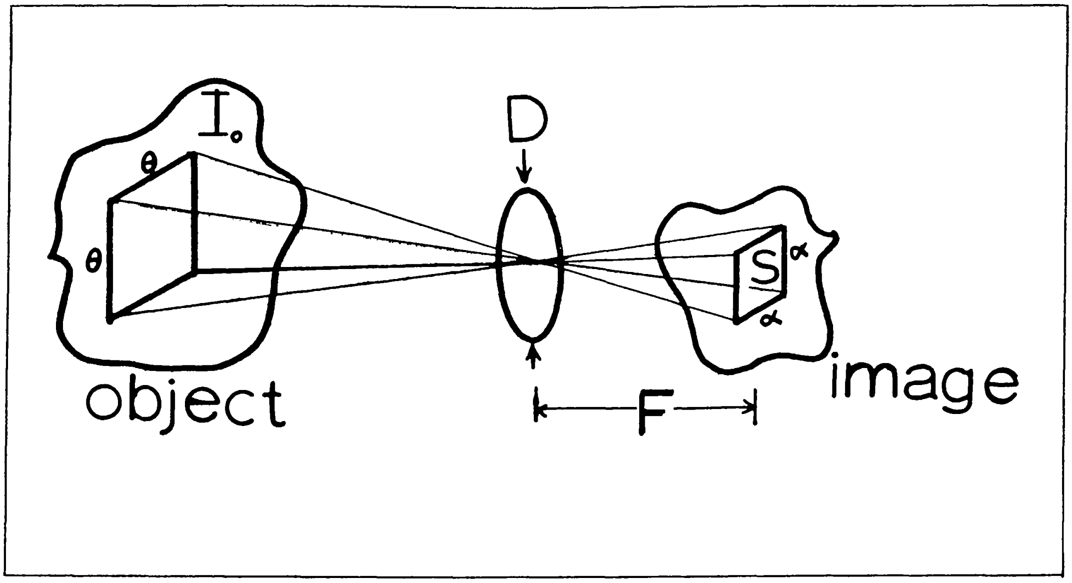

Figure 1. Image of an extended object. The square α by α is a resolution element of the detector.

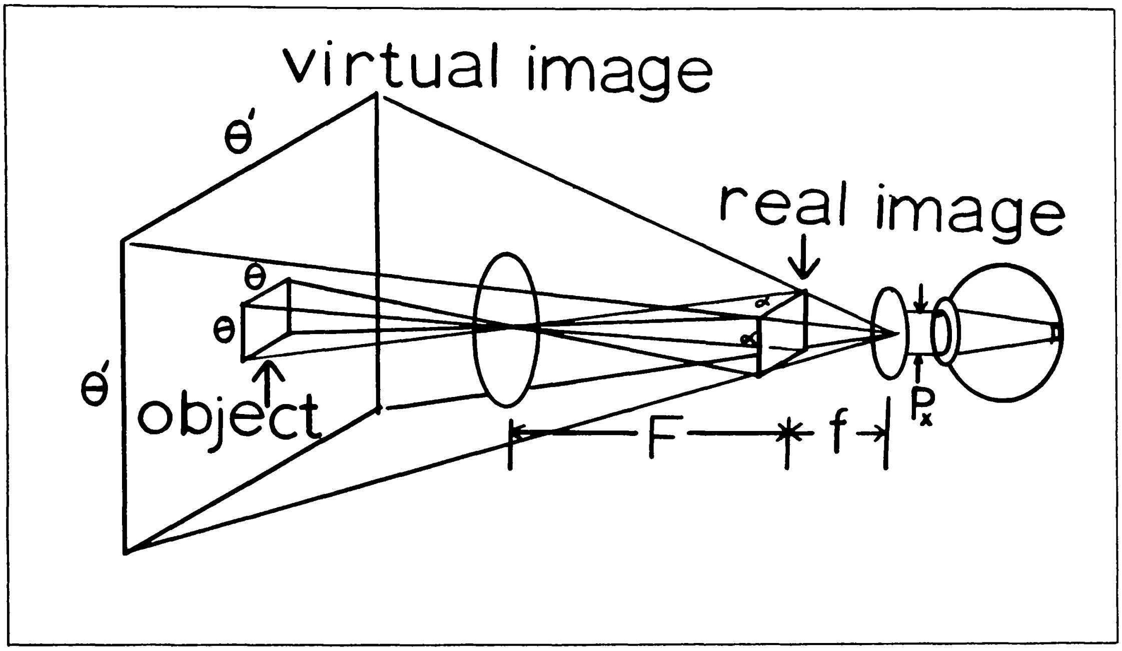

Figure 2. Image of an extended object viewed visually. Resolution element α by α projects onto the sky as square θ' by θ'.

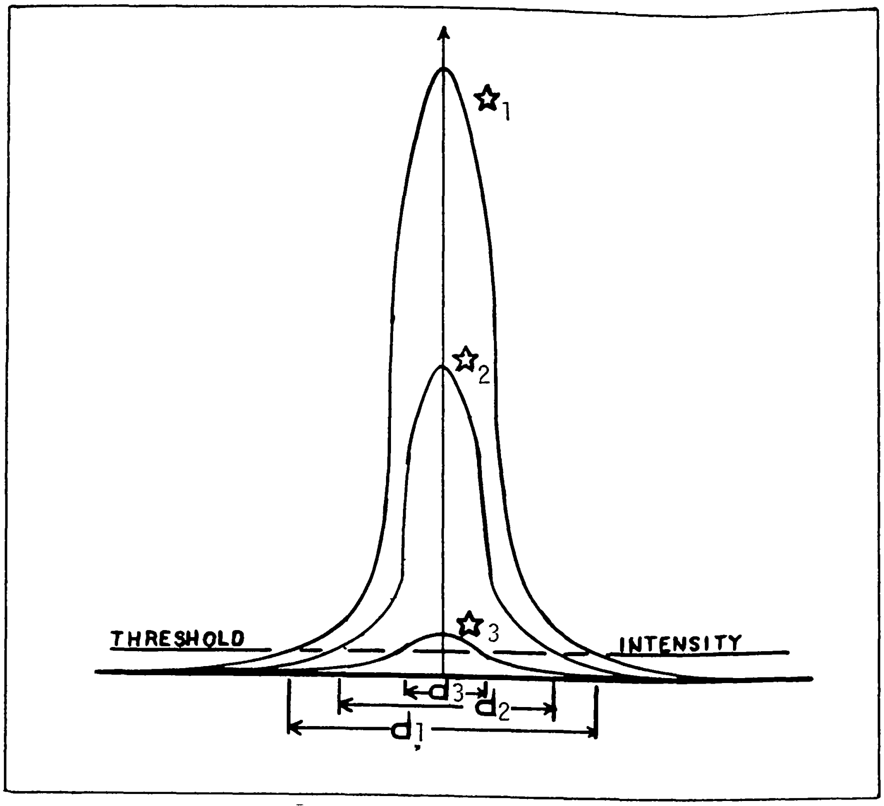

Figure 3. Intensity profiles of three stars of different magnitude. The apparent diameter of each is determined by the threshold of visibility of the eye.

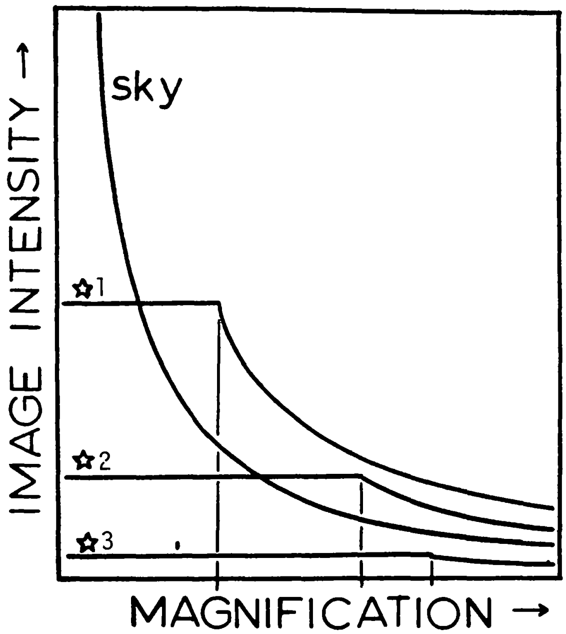

Figure 4. Apparent intensity of the sky and three stars seen through an eyepiece vs. magnification. While the sky intensity continually decreases, a star will remain constant until its apparent diameter exceeds the limit of visual resolution. Magnification beyond this point will not increase contrast.