Low Resolution Spectroscopy of Miras III—R Centauri

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Abstract In recent years low resolution spectroscopy has become available for use in crowd-sourced campaigns. One type of target that could benefit from a crowd-sourced photometry and spectroscopy campaign is Mira-type variables undergoing significant changes in their light curves in any combination of amplitude, average luminosity, period, and shape. These types of Miras have been previously studied and are hypothesized to be undergoing a helium shell flash. Previously published spectroscopic studies showed that R Cen, a dual-maximum Mira undergoing significant changes to its light curve, cycled between a spectral type of about M4.5 at maximum light to M9+ at the deeper of the two minima. Our results, from 63 spectra taken over the course of a full pulsation cycle, show that the spectral type at maximum light is still about M4.5, while at minimum light the spectrum now only goes as late as between M6 and M7. This lends support to the hypothesis that R Cen is undergoing a shell flash and is in the increasing energy output phase of the flash cycle. Some helium shell flashes result in a dredge-up of thermonuclear-processed materials from the deep interior into the stellar atmosphere. Our results show that major changes in spectral type from M to S to C can be detected with very modest observing equipment. The ability to detect both types of changes shows that low resolution spectroscopy, which can be undertaken by amateurs, clearly has scientific value.

1. Introduction

Low cost, filter wheel gratings such as Paton Hawksley SA-100 and SA-200 allow non-professional observers to do low resolution spectroscopy of bright targets as easily as they take a CCD image. The resolving power of these gratings varies from $R = \lambda/\delta\lambda \sim 100 - 500$, depending on the grating used, the distance between the filter-wheel and the imaging plane, and the focal ratio of the telescope. While sharp spectral features are blurred at this resolution, the gross spectral features including spectral type and shape of the underlying blackbody curve can be measured. The processing of learning MK spectral classification can be laborious, so we have sought to develop quantitative methods to measure filter-wheel grating spectra to make them easier to incorporate into on-going crowd-sourced AAVSO observing campaigns (Martin *et al.* 2016).

Their large amplitudes and long periods make Mira-type variables a favorite target of non-professional observers (AAVSO LPV Section). A typical non-professional filter-wheel grating setup, like that used in this work, will be limited to stars that are brighter than magnitude 12 in Johnson V. A search of Mira variables in SIMBAD (Wenger *et al.* 2000) returns 1316 Miras and 614 S-types brighter than magnitude 12 (with overlap between the lists). Among the large numbers of known Miras are a small subset which are undergoing significant changes in their light curve in any combination of amplitude, average luminosity, period, and shape. These changes in Mira light curves are conjectured to be the effects of a helium shell flash (Templeton *et al.* 2005). A helium shell flash occurs when a shell of inert helium overlaying a carbon core reaches the required temperature and pressure to ignite and there is a rapid spread of thermonuclear burning throughout the helium layer.

Numerical modelling, for example by Wood and Zarro (1981), indicates that at the onset of a helium flash the energy production of the star, and hence its luminosity, undergoes an initial sharp drop because the flash disrupts the energy production in the hydrogen burning shell immediately above it. However, the heat generated by the flash eventually reaches the surface, increasing the luminosity to a peak before gradually decreasing as the helium is fused and energy production reverts to being solely from hydrogen shell burning.

A second effect of a helium shell flash is that it may create a convection zone which runs from the thermonuclear burning shell to the stellar surface. This brings up recently processed nuclear materials to the surface and is referred to as a dredge-up event. The consequence is that freshly fused carbon may be transported to the stellar atmosphere altering the atmospheric chemistry and hence its spectrum.

A further finding of numerical modelling (Wood and Zarro 1981) is that the time between the onset of one helium flash, which models suggest lasts 100–500 years, through the quiescent hydrogen burning phase to the onset of the next helium flash is on the order of 50,000 years. Thus, to gather observational evidence to discriminate among competing models requires a statistical approach of observing a sufficiently large number of Miras that some of the sample are experiencing the effects of a shell flash, and continuing these observations over a very long time period.

The bright southern hemisphere Mira, R Cen, is a Mira which has undergone significant changes in its pulsation period and light curve shape, and to a lesser extent in its luminosity, thus making it a candidate Mira for currently experiencing the effects of a helium shell flash. The purpose of this paper is to show that our observations, made using a grating wheel spectrograph, provide supporting evidence in favor of the shell flash hypothesis.

The remainder of the paper is structured as follows. Section 2 discusses past observations of R Centauri from visual, photometric, and spectroscopic observations. Section 3 discusses some practical considerations relating to the telescope and CCD camera used in observing when using filter wheel grating spectrographs. Section 4 details the observing equipment used. Section 5 presents the observational results of using a filter wheel grating spectrograph to observe R Cen. Section 6 contains our discussion and section 7 our conclusions.

2. R Centauri

RCen ($\alpha = 14^{h}\ 16^{m}\ 35.6s$, $\delta = -59°\ 54'\ 43''$ J2000) is a Mira type variable with a primary period around 500 days (Vogt *et al.* 2016; Templeton *et al.* 2005; Garcia *et al.* 2006; Walker and Greaves 2001). It is a favorite of southern hemisphere observers because it is circumpolar and can be observed year round from middle southern latitudes. Since 1891 observers have reported more than 30,000 brightness measurements of R Cen to the AAVSO International Database (Kafka 2019).

R Cen was the prototype for dual maximum Mira variables, which exhibit two maxima of approximately the equal brightness together with alternating deep and shallow minima (Walker and Greaves 2001). Dual maximum Mira variables are rare, suggesting that they represent a relatively brief and rapidly evolving stage of Mira evolution (Hawkins *et al.* 2001). Over the last 70 years, R Cen's light curve has significantly changed in appearance. Figure 1 shows a plot of AAVSO data from 1941 to 1954 compared with 1995 to 2009. Starting in the early 1980's the secondary peak started to fade relative to the primary (Walker and Greaves 2001). In the most recent cycles, the second maximum has been no more than a small rise in brightness on its descending phase and the minima are now almost two magnitudes brighter than observed in the mid-20th century.

During the years 1979–1982, the primary and secondary peaks were roughly equal in brightness, and Crowe (1984) measured R Cen's variation of spectral type from M5.5 at maximum brightness to M9+ at minimum. There was a high correlation between the brightness of the star, the strength of Ca I absorption in the spectrum, and the spectral type. At that time both peaks had roughly the same spectral type. Unfortunately, Crowe's observations did not provide good coverage of the primary maximum. Spectra were taken 80 days before and 140 days after the primary maximum in 1980 and 150 days before and 40 days after the primary maximum in 1981. But spectra were taken within 10 days of the secondary maximum in 1980 and 1982. While there was not a clear difference in spectral type detected in either maximum, the Balmer emission in the spectrum was more prominent at the time of primary maximum than it was at the secondary maximum. Over the same time period Walker and Greaves (2001) reported that the (B–V) color of R Cen was bluest around the time of primary maximum and much redder during secondary maximum.

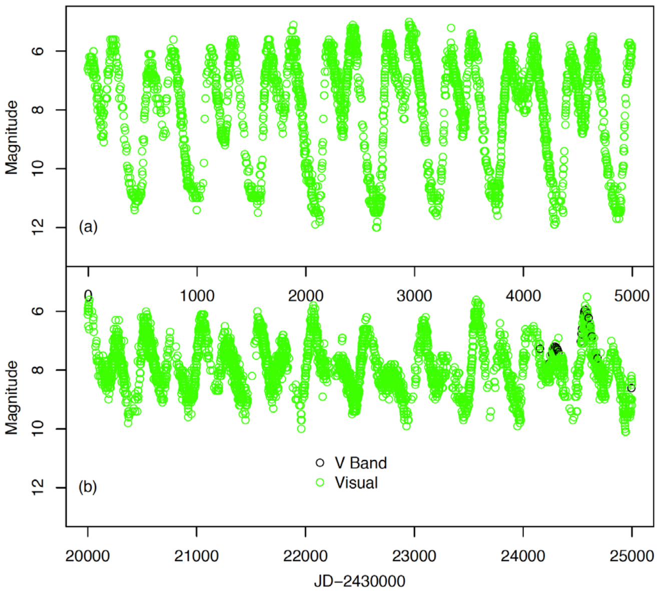

Figure 1. The light curve of R Centauri for two separate periods of 5000 days. In panel (a) the light curve 1941–1954 shows two clear maxima of nearly equal brightness. Panel (b) shows the light curve from 1995–2009 after the period had shortened, the amplitude decreased, and the second maximum reduced to a small rise in the descending phase.

The influence of prominent absorption bands on the spectral flux of late type giants makes (B–V) an ambiguous indicator. Spectroscopy is the preferred diagnostic tool. In 1968, Keenan *et al.* (1974) observed an earlier spectral type within a month of the primary maximum (M4.5) than Crowe (1984) measured a decade later (M6). It has not been resolved if this was a genuine change due to evolution of the star or caused by differences in either measurement of time relative to the maximum or measurement methods. These professionally obtained spectra were widely spaced in time. Given that low resolution filter-wheel grating spectroscopy can track changes around the cycles of Mira variables (Rea 2019; Rea and Martin 2021), they can be effectively employed at much higher cadence. Indeed, these methods applied to R Cen should provide a good comparison of the spectral types exhibited around its cycle now relative to what Crowe (1984) observed 40 years ago.

In addition to changes in the relative brightness of the primary and secondary peaks, Walker and Greaves (2001) noted the primary period of R Cen was growing shorter. Hawkins *et al.* (2001) reported that using a power spectrum method the period was mostly stable 1930–1966, while wavelet methods showed that the period decreased at a rate of –1 day/year since 1951. Using a longer baseline, Templeton *et al.* (2005) reported an average rate of change half that, but acknowledged suspicions by Walker and Greaves (2001 and Hawkins *et al.* (2001) that the rate of period change had probably accelerated in the second half of the twentieth century.

Together, the changes in light curve shape and period indicate that R Cen could be undergoing rapid evolutionary change. R Cen is one of eight (8) out of 547 Mira variables (1.5%) Templeton *et al.* (2005) analyzed which exhibit period changes with a high degree of confidence. Templeton *et al.* (2005) argue that the period changes of those Miras could be due to the effects of thermal pulses, of which a helium shell flash in one possibility. Theoretical models by Vassiliadis and Wood (1993) predict that a similar percentage (2–3%) of Mira variables should be undergoing thermal pulses at any given time. Thermal pulses may result in a dredge-up event which brings thermonuclearly processed material from deep inside the star to the surface. Over a few decades this phenomenon transformed the spectrum of Mira variable BH Cru from SC to C (Uttenthaler 2013).

Low resolution filter wheel grating spectroscopy should be sensitive to similar changes in R Cen (Martin *et al.* 2016).

3. Some practical considerations

Practical application of the results from the Pickles (1998) template spectra, detailed in Rea and Martin (2021), revealed several important caveats and considerations. In working around each issue, we sought to satisfy the goal to create a process that would be no more difficult to implement than differential photometry.

3.1. Wavelength calibration

One of the general challenges of spectroscopy is wavelength calibration. Usually, the largest source of uncertainty in the wavelength calibration is the uncertainty in the zero point. This is especially true for filter wheel grating spectra, which rely on measuring the position of the usually over-exposed zero-order image to set the zero point for the spectrum.

For most filter-wheel grating setups, the zero point of a spectrum can be determined reliably within fewer than ± 3 pixels. For the setup we were using, that correlated with ± 5 nm. To be conservative, we expanded the margin of uncertainty out to ± 7.5 nm.

The quasi-continuum peaks, listed in Table 1, are the local peaks in the spectrum. To account for uncertainty in the wavelength scale we measured the peak flux at the maximum flux value within ± 7.5 nm of the expected wavelength. The maximum TiO absorption is the local minimum. Therefore, we measured the TiO flux at the minimum flux value within ± 7.5 nm of 719 nm.

We measured our spectra with automated subroutines, so we needed to adopt a search range for the peak or minimum. But note that this approach allows a person familiar with the spectrum shape (and the location of the peaks) to measure the quasi-continuum points of TiO by eye without a formal wavelength calibration. This has a clear advantage making the process more accessible and easier to use.

3.2. Near-infrared CCD response

Silicon detectors have a well-known drop off in efficiency in the near-infrared part of the spectrum. M-type stars put out most their flux in the infrared, so the spectrum is recorded mostly in regime of declining efficiency on the CCD spectral response curve between 600 and 1000 nm. A typical CCD has 20% lower sensitivity at the C5 point (884 nm) (see Table 1) than at the C2 point (704 nm). Longer exposure and image stacking can help, but the C6 point (904 nm) is unusable in spectra recorded with most CCDs. To accommodate this, C6 was not used in any ratios within this paper.

Table 1. The six possible continuum ponts in spectral class K5 to M9 identied by Kirkpatrick *et al.* (1991).

Point	*Value (nm)*
C1	653
C2	704
C3	756
C4	813
C5	884
C6	904

3.3. Decreasing blue flux

As a Mira variable gets cooler and fainter the spectral flux shifts red-ward. For pulsating variables, their temperature is coolest when the variable is faintest, see, for example, Figure 1 of Rea and Martin (2021), which shows the spectrum of R Oct at three times, including maximum and minimum light. The practical implication of this is that at minimum light there will be a significant decrease in signal-to-noise (SNR) in the blue end of the spectrum. That has a clear effect on the C1 and C2 continuum points and on the TiO absorption line measurements.

Sometimes the SNR of the spectrum can be improved through longer exposures and stacking more images. But practically speaking, for the types of small telescopes which amateurs use, this may not be sufficient to produce high SNR spectra. Unfortunately, the poor SNR in the blue end of the spectrum has a clear effect on the C1 point when the star is the coolest and faintest. That has a direct effect on the quality of the C1/C3 ratio, which ought to be sensitive to differences in the latest/coolest spectral types.

3.4. Comparison stars

The final two considerations relate to calibrating the quasi-continuum ratios and Wing ratio across different telescopes and camera systems. The resolution of the spectrum depends on the grating used and the distance between the grating and the imaging plane. Figure 2 presents two spectra from Mira variables with the same spectral type and temperature taken with different telescopes/grating/camera setups. The difference between spectra is mostly due to the difference in resolution. At higher spectral resolution the peaks and troughs are sharper and at lower resolution they are more blurred. The spectral response of the CCD and telescope combination also affects the relative flux across the spectrum.

To address these issues, we introduced spectroscopic comparison stars observed with the target. Like photometric comparison stars, they were selected either for low spectral variability and/or to cover the range of spectral types which the target progresses through over its cycle. Finding suitable low amplitude comparison stars for very late Miras such as M8 or M9 often cannot be done and published ranges of higher amplitude Miras must be used. For example, Table (2) shows the C2/C5 ratio for five comparison stars that we observed along with R Cen.

4. Observing equipment

One telescope participated in this study. We operated an 80-mm f/6 Explore Scientific apochromatic refractor in Christchurch, New Zealand, with an Atik 414E Mono CCD camera using a SONY ICX424AL front-illuminated chip. The plate scale in the imaging plane was 2.77 arcseconds/pixel. We used a Paton Hawksley Star Analyzer 100 grating yielding a first order spectrum with a dispersion of 1.488 nm/pixel.

Table 2. The details of the variability in spectral type and magnitude of a selection of five comparison stars.

Star	*Spectral Type*	*V_{mag} Range*	*C2/C5*
R Oct	M5.3–8.4	6.4–13.2	1.04 ± 0.43
CV Oct	M3	8.9–9.2	3.07 ± 0.39
BQ Oct	M4III	6.8	3.36 ± 0.21
CQ Oct	M4/M5III	8.12–8.59	2.49 ± 0.20
BW Oct	M5-M7III	7.9–9.1	1.12 ± 0.06

Note: An expanded version of this table appears in Rea and Martin (2021) Table 2.

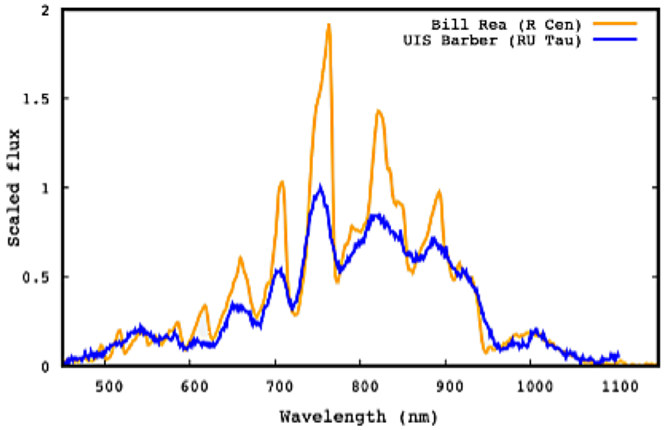

Figure 2. Two spectra of Mira variables of the same spectral type and effective temperature taken with two different telescope/grating/camera systems.

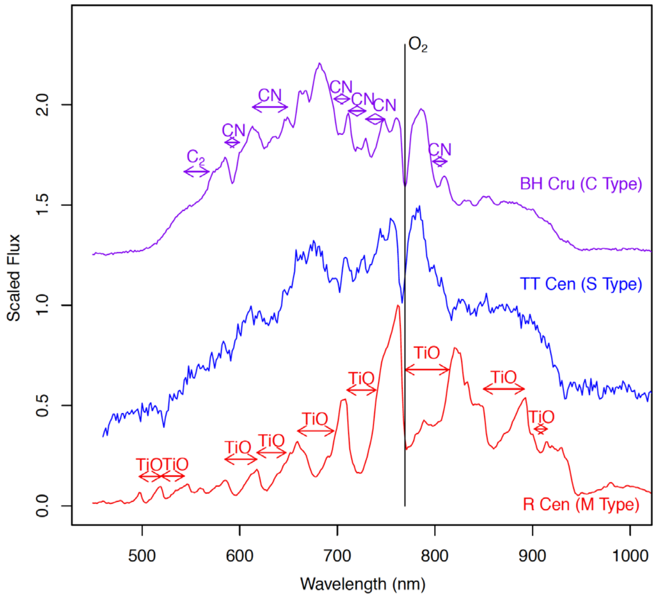

Figure 3. The spectra R Cen, TT Cen, and BH Cru, with spectral types M, S, and C, respectively. The spectra are offset on the vertical axis so that they do not overlap. The black vertical line labelled O_2 is caused by the oxygen molecules in the earth's atmosphere.

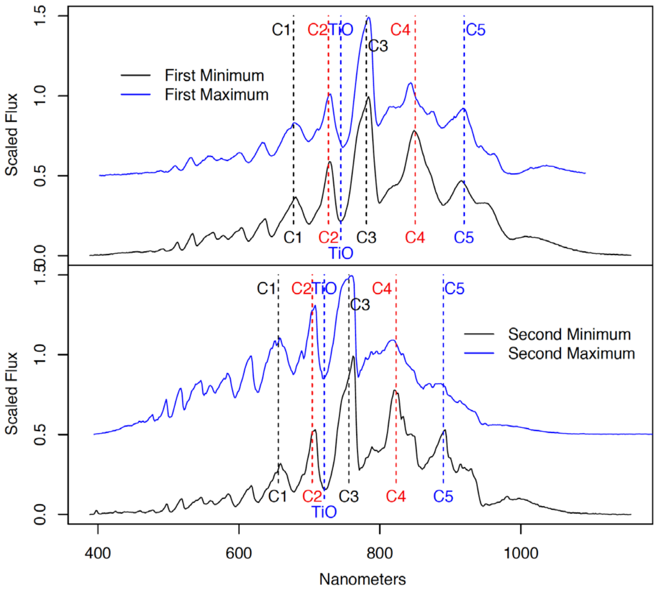

Figure 4. Four spectra of R Cen, an M-type Mira. Panel (a) close to the first maximum and minimum light, and panel (b) close to the second maximum and minimum. The spectra are offset on the vertical axis so that they do not overlap.

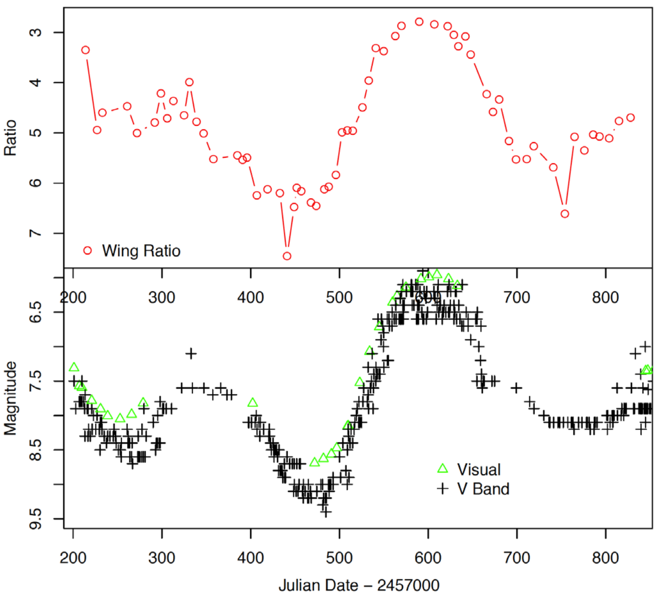

Figure 5. The Wing ratios (upper panel) and the light curve (lower panel) of R Cen, an M-type Mira, over the course of a pulsation cycle. The data source for the light curve is the AAVSO International Database (Kafka 2019).

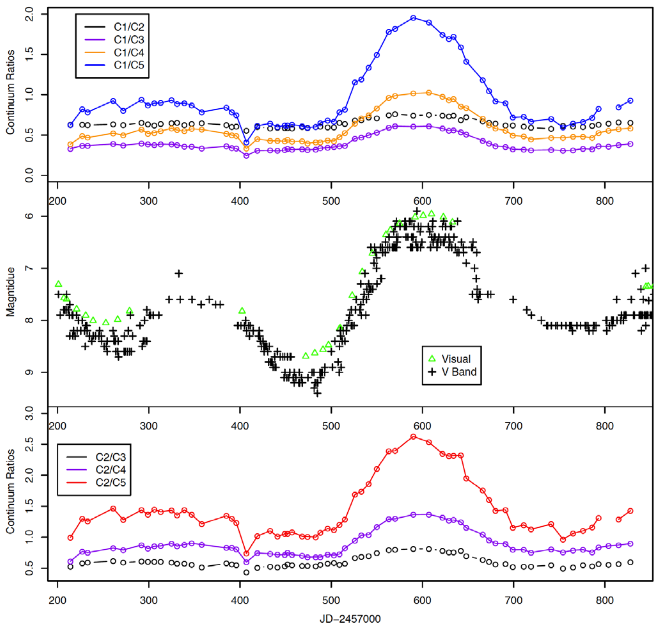

Figure 6. The continuum ratios of R Cen, an M-type Mira, for continuum points C1 (top panel) and C2 (bottom panel) together with its light curve (middle panel) over the course of a pulsation cycle.

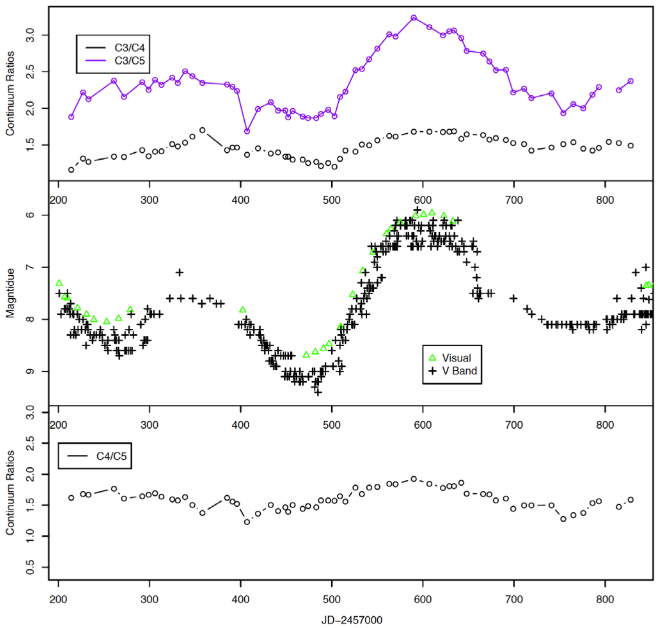

Figure 7. The continuum ratios of R Cen, an M-type Mira, for continuum points C3 (top panel) and C4 (bottom panel) and C5 together with its light curve (middle panel) over the course of a pulsation cycle.

5. Spectra

In this section we present the main spectral results. We first present the results of the observations of the M-, S-, and C-type Miras, then the results of spectral changes in R Cen owing to its pulsation.

Figure 3 presents three spectra of southern hemisphere Miras with spectral types M, S, and C, respectively, namely R Centauri, TT Centauri, and BH Crucis.

We obtained 63 spectra of R Cen over the course of a full pulsation cycle. Although evolving, it still has two identifiable maxima and minima over the course of a single pulsation cycle. Four spectra from these maxima and minima are presented in Figure 4. We also present the light curve and the Wing and a range of continuum ratios obtained in Figures 5 through 7. The Wing and continuum ratios were calculated as outlined in Rea (2019) and Rea and Martin (2021).

6. Discussion

Figure 3 allows us to address the question of whether a dredge-up event could be detected with a filter wheel spectrograph. Although the spectrum of TT Cen in Figure 3 is quite noisy because it was quite dim when the spectrum was obtained, it is clear that the M-, S-, and C-type Miras have distinctly different spectra even at these low resolutions. Thus, if a Mira was being observed at the time of a dredge up event of sufficient magnitude that the spectral type changed from M to S or S to C then this could be detected by visual inspection of the spectra and hence reported to professional observatories for follow up, without the need for more sophisticated analysis of the spectra.

Figure 5 addresses the question of whether the effective temperature and spectral type of a Mira can be reliably determined with a filter wheel grating spectroscope. This question was earlier addressed by Rea (2019) and Rea and Martin (2021). Those two papers showed that by using the strength of the TiO absorption band at approximately 719 nanometers relative to the strength of the continuum at about 756 nanometers, a ratio termed the Wing ratio because of its proposal by Wing (1992), is it possible to assign a spectral class in the range of M3 to about either M6 or M7. For cooler Miras the formation of a VO line makes the 756-nanometer continuum point unreliable and one then needs to switch to using continuum ratios.

The observed changes in R Cen are hypothesized to be the result of a helium flash. While a flash initially results in a significant decrease in energy output, over the course of 100 to 200 years after the initial decrease, the energy output slowly increases. Hence, should the hypothesis be correct and R Cen is in the brightening phase of the flash, we would expect that it is warmer now than when previous estimates of its spectral range were published. Unfortunately, those were from professionally obtained spectra and we do not have low resolution spectra from when R Cen was clearly a dual-maximum Mira to compare with our more recent spectra. However, using the ratios to spectral class figures given in Rea (2019) and Rea and Martin (2021), R Cen now appears to cycle between M4.5 and M6-M7.

This is significantly warmer than the M5.5–M9+ range reported by Crowe (1984). While the spectral type at maximum light of M4.5 is equal to that reported by Keenan *et al.* (1974), at minimum light the spectral type clearly does not go as late as M8, and certainly not to M9, as reported in the earlier literature. The formation of the VO line which makes the 756 nm continuum point unusable begins somewhere between M6 and M7 (see Figure 2 in Rea and Martin 2021) can only just be seen in the second minimum of R Cen's spectra in Figure 4). If R Cen was of type M8 or M9 at minimum light its spectra would resemble those of R Oct in Figure 1 of Rea and Martin (2021), which it clearly does not.

While these results give some support to the helium flash hypothesis, the flash, if it is occurring, appears to have had little to no effect on the spectral types at maximum; rather, the resulting changes in spectral type have had a much greater effect on the cooler parts of its cycle which are towards minimum light. It is not at all clear from the computational studies whether these are the expected changes.

The alternative to using the strength of designated absorption lines is to more directly measure the black body radiation curve, which in Miras is largely hidden by numerous molecular absorption lines. Nevertheless, in R Cen we can get an estimate of the strength of the continuum at five points, namely C1 through C5 in Table 1. We can form 10 ratios of these five points. These have been calculated and are presented in Figures 6 and 7. Of the 10 ratios, the two best for determining spectral changes appear to be C1/C5 (653/884 nm) and C2/C4 (704/884 nm), although C3/C5 (756/884 nm) also appears to give a good picture of the variation. The three ratios C1/C2, C1/C3, and C2/C3 present a consistent picture of the changes and there is little to choose among them.

It is quite curious that despite the obvious changes in brightness in the light curve, there is little variation in the continuum ratios apart from when the light curve comes up to maximum light. The Wing ratio changes seen in Figure 5 appears to be more informative than the continuum ratios in this case. Within the estimated range of spectral type over the pulsation cycle, from about M4/M4.5 to M6/M7, it is only at the latest types, which occur at the deep minimum, that the Wing ratio becomes unreliable. So while the continuum ratios are informative, R Cen's current spectral range does not go late enough to require their use.

In addition to the types of changes considered in this paper, Miras do have natural variability in their periods, amplitudes of their light curves, and the brightness of their maxima and minima. These changes, particularly changes in the brightness at maximum and minimum, may be reflected in the variability of their spectral type. There are two immediate causes of variation in the brightness, namely radius and temperature, in which a larger radius gives a brighter star for a fixed temperature and a higher temperature gives a brighter star for a fixed radius. Low resolution spectra should be of scientific value in investigating these types of changes, and some previous work on demonstrating that is presented in Rea and Martin (2021) Figure 3 from a theoretical point of view and Figure 4 from an observational point of view.

7. Conclusions

Figure 3 shows that the evolution in spectral type M → S → C is easily detectable by grating spectroscopes. Hence the idea of monitoring a significant number of Miras experiencing thermal pulses in order to observe these types of large spectral changes appears technically feasible. However, with only two clear cases and one uncertain case (see Whitelock (1999) and Uttenthaler *et al.* (2016) for details) in the last 50 years, such events are rare. The observations which were made gave no indication of how quickly the spectral type changed during the dredge-up event. This is an area of research which would benefit from long-term, high cadence spectroscopy by dedicated amateurs.

There are clear changes observable in the spectra with temperature in the atmosphere of R Cen with the modest equipment we were using. This could be seen over the course of the cycle we observed and the scientific value of such observations can be seen in the spectra gathered for R Cen.

Of particular interest is the fact that the hypothesis that R Cen is experiencing a helium flash was supported by our observations. Unexpectedly, the magnitude and spectral type at maximum light does not appear to have changed much, if at all, since the observations by Keenan *et al.* (1974). The visual observations in the AAVSO International Data indicate about magnitude 6 at maximum light while minimum light has increased by about two magnitudes from 12 to 10. The spectral type at minimum light increased from M9+ as observed by Crowe (1984) to about M6/M7 in our observations. Both the increase in brightness and the increase in temperature implied by the spectral changes indicate a clear increase in energy output by R Cen during this period of time. These observations have given a clear example of the scientific value of routine monitoring of Miras using low resolution spectroscopy. With a number of candidates for Miras undergoing a helium flash already identified by Templeton *et al.* (2005), interested observers should have no trouble in selecting targets to suit their particular setups. Templeton *et al.* (2005) used changes in the light curves to identify the candidates. For Miras which have been observed at high spectral resolution the presence of technetium (Tc) in the spectrum would indicate that a dredge-up event had occurred or was in progress and these stars could also be added to an observing program.

8. Acknowledgements

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