An Undergraduate Research Experience on Studying Variable Stars

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Abstract We describe and evaluate a summer undergraduate research project and experience by one of us (AA), under the supervision of the other (JP). The aim of the project was to sample current approaches to analyzing variable star data, and topics related to the study of Mira variable stars and their astrophysical importance. This project was done through the Summer Undergraduate Research Program (SURP) in astronomy at the University of Toronto. SURP allowed undergraduate students to explore and learn about many topics within astronomy and astrophysics, from instrumentation to cosmology. SURP introduced students to key skills which are essential for students hoping to pursue graduate studies in any scientific field. Variable stars proved to be an excellent topic for a research project. For beginners to independent research, it introduces key concepts in research such as critical thinking and problem solving, while illuminating previously learned topics in stellar physics. The focus of this summer project was to compare observations with structural and evolutionary models, including modelling the random walk behavior exhibited in the (O–C) diagrams of most Mira stars. We found that the random walk could be modelled by using random fluctuations of the period. This explanation agreed well with observations.

1. Introduction

Variable star analysis is an excellent way for students to develop and integrate their science, math, and computing skills, motivated by the knowledge that they are doing real science, with real data (Percy 2008). This paper describes a summer undergraduate research experience by co-author Amaral, supervised by co-author Percy, within the context of a formal program in which students' research is supplemented by a "professional development" program.

In the past, Percy has assigned students a small, self-contained research project which will lead to a publishable paper with the student as co-author (e.g. Percy and Huang 2015). In the present case, there were two significant differences. First: Amaral had been exposed, in a broad way, to the nature and evolution of Mira stars in a third-year course in astrophysics, and was specifically interested in becoming familiar with a variety of ways of using observations to understand Mira stars. Second: this project was carried out within a formal educational framework. We are aware that this *Journal* wishes to feature more papers in the area of science education. Such papers are most useful if they include a description and evaluation of the educational project or activity, for the benefit of readers who organize or participate in such projects and activities, or who may wish to become involved with them.

We therefore begin with a summary of the Mira star project, which includes three parts: the reliable identification of double-mode pulsators, and the interpretation of (O–C) diagrams in terms of evolution, and of random fluctuations. We then describe and evaluate the overall summer experience.

2. Summer project

Author Amaral, having learned about Mira stars and their evolution in a third-year astrophysics course, decided that she wanted to sample various ways of comparing Mira star observations with models of their structure and evolution. That led to the following sub-projects.

2.1. Identifying bimodal pulsators

Variable star data usually come in the form of light curves, which often extend as far back as a century. Much of this data, especially for Mira stars, is collected by skilled amateur astronomers. This is necessary because there are many variable stars, and observing their changes in magnitude regularly over many years is extremely time-consuming—too much so for professional astronomers. To analyze the massive amount of data on these stars, time series analysis is necessary. Specifically for this section of the project, we used Fourier analysis. Fourier analysis attempts to fit a sum of sinusoids to the light curve. It scans a pre-chosen range of periods, at fine resolution. If a sinusoid with a particular period fits the data, then we can say that period is present. Fourier analysis thus generates a power spectrum, which picks up any periodic variations in the light curve. The peaks in the power spectrum represent possible periods of pulsation of the variable star.

Many pulsating red giant stars pulsate in two or more modes. This effect can be found by looking at the power spectrum and selecting and evaluating the prominent peaks. Fourier analysis makes it very easy to detect the modes of pulsation, though this process comes with many challenges. The power spectrum can contain false periods, namely, alias, spurious, and harmonic periods. One-year alias periods can be detected as peaks in the power spectrum which differ in frequency from the real periods by 1/365.25 cycle per day. They are caused by the fact that some stars are only visible at certain times of the year, thus creating a periodicity in the obtaining of data. Spurious periods are periods of exactly one year found in the power spectrum. Finally, harmonic periods are a result of the Fourier transform fitting sinusoids to a non-sinusoidal light curve. It is common to find incorrect multi-modal stars identified in the literature in cases where the power spectrum was not carefully analyzed.

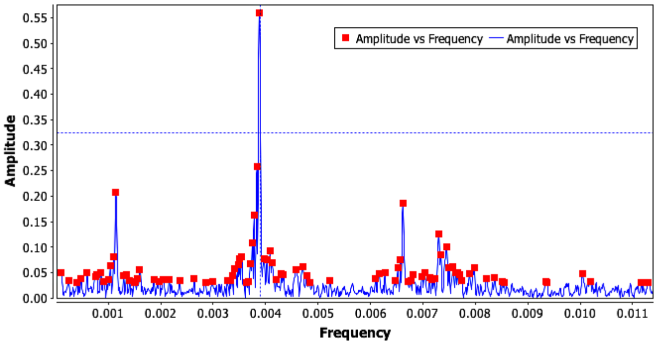

Figure 1. The amplitude spectrum of V Boo. The strongest peak is at 257.7 days, or 0.00388 cycle/day. The peaks at 0.00114 and 0.00662 cycle/day are yearly aliases of this. The peak at 0.00730 cycle/day or 137.0 days is a true secondary period.

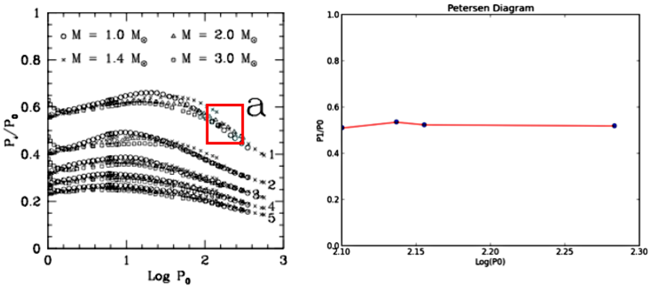

Figure 2. The figure on the left is a theoretically derived Petersen Diagram from Xiong and Deng (2007) for various assumed masses and overtones. We compare this to the figure on the right which is a Petersen diagram created using the bimodal stars detected. This figure represents the area on the Xiong and Deng (2007) diagram that is within the red square.

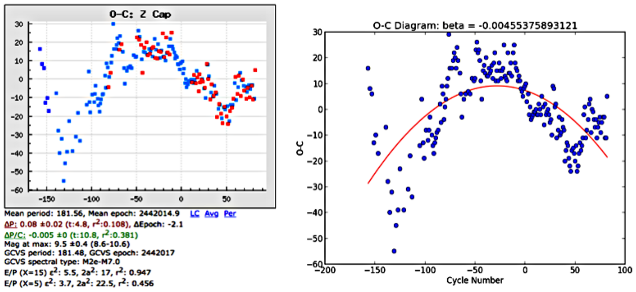

Figure 3. The figure on the left shows the O–C diagram for Z Cap (from http://var.astronet.se/mirainfooc.php). The figure on the right is an example of fitting a parabola to the exact same data and obtain β which is the rate of period change. Notice the parabolic shape.

Being able to locate multi-modal pulsators accurately is extremely useful when comparing with theoretical structural models, which can provide theoretical periods and period ratios. Petersen diagrams are graphs of period ratios plotted against the log of the longer periods. Xiong and Deng (2007) have developed structural and pulsational models for red giants which use an improved treatment of convection, and assuming non-adiabatic oscillations, and have used these to make theoretical Petersen diagrams. By properly detecting bimodal stars, observed period ratios can be compared with the theoretical ones produced by Xiong and Deng.

For this part of the project, we used visual observations of pulsating red giants from the AAVSO International Database (Kafka 2015), and analyzed them with the AAVSO VSTAR time series analysis software (Benn 2013). We examined the following stars which showed possible evidence of bimodal behavior: ST And, V Aqr, V Boo, WZ Cas, W Cyg, U LMi, BQ Ori, and Y Tau, looking carefully for periods which were not spurious. We consider the following four stars to be bimodal; P0, P1 (both in days) and P1/P0 are given: V Boo: 256, 137, 0.535; WZ Cas: 370, 192, 0.519; U LMi: 273, 143, 0.524; BQ Ori: 247, 126, 0.510. These period ratios are consistent with the theoretical period ratios found in Xiong and Deng (2007) for fundamental and first-overtone pulsation.

Using the four stars considered for period analysis above we produced a Petersen diagram to compare to the theoretically derived Petersen diagrams found in Xiong and Deng (2007). Comparing observed Petersen diagrams to theoretical ones allows for a direct test of the structural theory used to derive the modelled Petersen diagrams. Such a comparison is also useful to determine the secondary period of a suspected bimodal pulsator if the main period of pulsation is known as well as the stellar mass. As can be seen from Figure (2), using only four sample stars to generate a Petersen diagram results in only a fraction of the theoretical curve to be covered; however, the reader can observe how important an accurate detection of the periods of bimodal stars can be in such a comparison.

2.2. Rates of period changes

Due to the fact that Mira stars have been observed and analyzed continually for up to a century—or longer in some cases—it is a well documented phenomenon that the periods of pulsations of many Mira stars tend to change. These changes in period can be classified by specific features, such as sudden changes, continuous changes (over a long period of time), and meandering changes. The best way to analyze any changes in the period is to use an (O–C) diagram; see Figure (3) on the left for an example. From a light curve one can measure times of maximum brightness, and generate an (O-C) diagram. (O–C) stands for "Observed" minus "Calculated," and can be generated by taking the observed times of maxima from the data and subtracting from them the calculated times of maxima, assuming the period to be constant. Due to the method used to calculate (O–C) diagrams, any linear changes in period will show up as parabolic on the diagram. A parabola opening upward will signify a positive period change, and a downward opening parabola will signify a negative period change. An (O–C) diagram allows for smaller changes in the period to be more detectable, because small changes in period have a cumulative effect on the times of maxima. (O–C) diagrams can also be used to calculate the rate of period change for the set of data. The average rate of period change can be determined by fitting a quadratic equation to the (O–C) diagram. The coefficient of the quadratic term is given by Equation (1), where K_0 is the quadratic coefficient and $\overline{P}$ is the average period over the time interval. A quadratic fit is used because, as mentioned above, linear changes in period show up as parabolic changes in the (O–C) diagrams, see Figure (3) on the right.

$$K_0 = \frac{\overline{P}}{2}\,\frac{dP}{dt} \qquad (1)$$

By using Equation (1) it is possible, in principle, to determine the evolutionary behavior of each star. Mira stars are slowly moving up the asymptotic giant branch (AGB) in the H-R diagram, and as such a slow period change due to this movement is expected, as well as very rapid period changes due to occasional thermal pulses (and possibly other factors such as extreme mass loss, super winds, and shock waves). By determining the amount of period change, it is possible to determine what physical processes are happening within the star.

Stars on the AGB have a degenerate core of carbon contained within a shell of helium burning which is surrounded by a shell of hydrogen burning. Once the helium-burning shell is burnt up, the fusion source will be provided solely by the hydrogen-burning layer. The helium will be fusing unstably for a long period of time, causing periods of intense burning when the helium-fusing layer is reignited. These periods of intense helium-shell burning are called thermal pulses. The majority of the time on the AGB is spent during the inter-pulse quiescent phases. During the thermal pulse the luminosity of the star peaks at a maximum and then decreases to less than it was before the pulse (Wood and Zarro 1981). A thermal pulse is expected to last about 10^3 years, therefore a star only spends about 1–2 percent of its time on the AGB in a thermal pulse phase (Wood and Zarro 1981). The thermal pulse is expected to drastically affect the period of the star. It is hypothesized that perhaps the Mira stars which exhibit the highest rate of period change are those which have had a recent thermal pulse or which are currently undergoing a thermal pulse. As mentioned previously, the value $\beta = \frac{dP}{dt}$ represents the *average* period change occurring, though when attempting to determine evolutionary properties it can be difficult because β includes period changes due to "random walk fluctuations" (see section 2.3). Karlsson (2014) attempted to deal with this issue by averaging the β of 362 stars in an attempt to average out these supposedly random fluctuations. Values of β, and other information about these stars is contained on his website. Templeton *et al.* (2005) have determined values of β for 547 Miras, using wavelet analysis.

2.3. Random walk fluctuations

Stemming from the discussion of period changes in section 2.2, we attempt to discuss the random walk behavior mentioned previously which has been observed in the majority of Mira stars. Random walks show up as meandering or winding changes in the period over time. They are on too short a time-scale to be due to a thermal pulse, or other known evolutionary factors. Since Mira stars can be found on the AGB, and they come from medium to low mass progenitors, they contain *extremely* large convective cells. Theoretical calculations done by Schwarzschild (1975) have estimated that only around a dozen convection cells would exist on the surface at one period of time; compare this to the millions on the surface of the Sun. Convection is a branch of stellar physics which is not as well understood, mostly due to the fact that convective turbulence is extremely difficult to model. These extremely large convection cells would thus have a significant impact on the processes occurring within the star, and add a factor of randomness to the periods of Miras. In attempt to quantify this effect, we used a previously calculated estimate done by Percy and Colivas (1999) who used the Eddington and Plakidis (1929) formalism. By using 391 Miras and determining the amount by which they fluctuate from their main pulsation period, we were able to take the approximately average value of ±0.2 percent change in period, per period, to model this random effect.

Using this information, it is possible to develop a simulation which can show the effect of this 0.2 percent random fluctuation on the pulsation period of Mira variable stars over time. Though Miras can sometimes be bimodal pulsators (usually pulsating in the fundamental and first overtone), the value used for this simulation can be approximated to just the main pulsation period. This simulation can be done quite easily using Python, using the RANDOM library to generate these random fluctuations. The Python RANDOM library uses the Mersenne Twister algorithm to generate random float values within a given range. The random fluctuation can be taken into account by using the recursive formula in Equation (2), where K_{rand} is a random number determined with 0.2 percent fluctuation.

$$P_{n+1} = P_n + K_{rand} \tag{2}$$

See the right side of Figure (4) for an example of period-versus-time graph generated using the method described above, and beneath it, the corresponding (O–C) diagram. By changing the period-versus-time graphs to (O–C) diagrams we were able to compare the generated (O–C) diagrams to (O–C) diagrams from actual Mira stars. We used the (O–C) diagrams provided by Karlsson (http://var.astronet.se/mirainfooc.php) to compare with our simulated diagrams. By doing this we found that we could generate similar (O–C) curves to the ones found in Karlsson's database. See Figure (4) for the (O–C) diagram of T Gru, which is being compared to the (O–C) diagrams generated by our simulation. Sometimes changing the 0.2 percent fluctuation rate was necessary (since 0.2 percent represents the average fluctuation value in the range found by Percy and Colivas). By changing the fluctuation amount and comparing to observed data we could determine which stars were more heavily influenced by these random fluctuations. By being able to approximately recreate (O–C) diagrams of Mira stars with our simulation we can conclude that these random walk fluctuations observed in many Mira stars (which could be caused by the random motions of convective cells) in the period can indeed be modelled and explained by random period fluctuations.

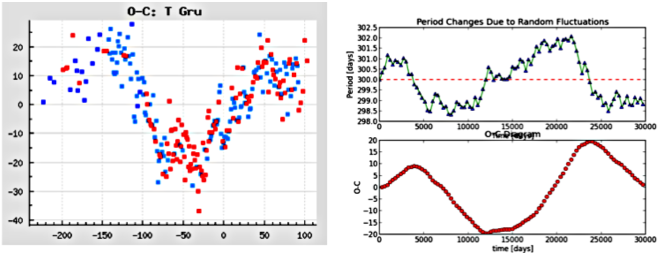

Figure 4. The figure on the left shows the (O–C) diagram for T Gru; notice its meandering features. The upper figure on the right is a simulated period-versus-time graph and the lower figure on the right is its corresponding (O–C) diagram. The simulation can be compared to the (O–C) diagram of T Gru.

2.4. Educational benefits

The project as a whole had many educational benefits, such as working with a research supervisor, planning a project, reading research papers (applying what was learned in the SURP workshops—see section 3), writing this report and giving a presentation, graphical and statistical concepts, and learning a lot about a specific topic—Mira stars. As outlined in section 2, this summer project focused mainly on pulsating red giant stars such as Miras and semiregular variables, in which we compared observations to models to determine key structural and evolutionary information. Coming into this summer with no prior experience in research, the topic of variable stars was able to serve as the perfect introduction to research, by applying the knowledge acquired in previous astronomy courses on stellar structure and evolution to this summer project. However, each section of the project led to many other specific educational benefits that allowed many wide-ranging skills to be developed that could be used in further astrophysical research.

This project served as an introduction to research from which skills were developed in order to deal with imperfect data. By learning how to use previously existing software (VSTAR), in section 2.1, we were able to apply the method of Fourier analysis to determine periods. The *false* periods identified in this section were an example of artefacts, which often arise when dealing with imperfect data. Both the skills learned from dealing with the artefacts which were a result from imperfect data, and the methods to analyze these data (time series analysis and Fourier transforms) are widely applicable in other areas of astrophysical research and in the sciences more generally.

Connecting and comparing observation to theory is an important part of research and this sort of analysis was ongoing within the project. For example, in section 2.1, by using Petersen diagrams created using observational data we were able to compare to theoretically determined Petersen diagrams (and hence the structural models in which they were derived). In addition to this, in section 2.2 we were able to use the rate of period change to determine what evolutionary state the star was in and therefore determine more about how Mira stars evolve and their overall place in stellar evolution.

Dealing with imperfect data is an important skill to master when doing research within any scientific field. In section 2.2 of the project we were able to learn the importance of curve fitting to imperfect data, and how curve fitting can lead to important scientific results. As section 2.2 focuses on understanding ways of measuring period changes, an important property present when studying Mira variable stars, we used curve-fitting techniques to determine the rates of period changes of the stars analyzed. The two methods that were used to determine the period changes were wavelet analysis and (O–C) diagrams. These two methods gave us ways to visualize changes with time and how to measure these effects. Understanding how and why both of these techniques work is important, as they have a much broader application within other fields of stellar astronomy, and build upon using curve fitting to determine the values of period changes.

Section 2.3 of the project allowed for use of custom-written code, whereas the previous parts of the project required only the use of VSTAR (pre-existing software). Much of research in astronomy (and other related scientific fields) relies heavily on the use of computer programming to analyze data. Writing code (using Python) to perform simulations is a great way to improve computer programming skills.

Simulations are important in many areas of astronomy, therefore by being able to incorporate these simple simulations in this project one can learn more about the subject (Mira stars and their period fluctuations), and that the fluctuations observed can be reproduced using these random fluctuation simulations, see section 2.3. This gives us insight and understanding as to the powerful role simulations play in studying astronomy as they can be used to describe astronomical phenomena and lead to deeper understanding about the subjects.

Variable star astronomy introduces some important topics that are essential for most research conducted in physics, such as Fourier transforms, polynomial fitting, and time series analysis. Therefore, the topic of variable stars served as a perfect introductory research topic, due to the fact that much of the required physics for understanding it is taken in courses previous to third year undergraduate level. It serves as an opportunity for astronomy or physics students to apply much of the knowledge learned in their courses from first and second year to a research problem while learning important skills and gaining research experience.

3. Summer Undergraduate Research Program

The annual Summer Undergraduate Research Program (SURP) at the University of Toronto is jointly hosted by the Department of Astronomy and Astrophysics (DAA), the Canadian Institute for Theoretical Astrophysics (CITA), and the Dunlap Institute (DI). Here we provide a description of the SURP 2015 program along with thoughts and comments. SURP was very capably organized by a committee of postdocs; see Acknowledgements for their names. SURP in 2015 ran from May 4 until August 17. It allowed undergraduate students from across Canada to conduct independent research under the direct supervision of a faculty member in DAA, CITA, or DI. Over the four months, students worked on projects from all areas of astrophysics, ranging from instrumentation to cosmology. Since many undergraduates are new to conducting independent research, SURP focused a lot on building necessary skills that would be useful for students hoping to pursue a career in any area of research. In fact, many of these skills could be applied to a wide variety of careers.

The program began with an intensive two-week crash course in computing, which introduced students to the basics of Python using Unix terminals to execute commands. This was extremely useful, especially to some of the younger students who had not yet had the opportunity to use Python. The second week of the crash course was exclusively for those conducting research in CITA, though it would have been beneficial for all SURP students. A useful addition to the research program would have been an "introduction to research" workshop which could cover topics such as how to prepare for a research project, what to expect throughout the summer, what makes good research etc.

The program was structured so that research was conducted every weekday. On Tuesdays and Thursdays, the whole group of summer students met together for one hour and discussed the progress they had made in their projects. Along with weekly meetings, special workshops were also held to help students learn essential skills that are necessary for foundations of research. The workshops offered were: inquiry-based learning, how to write an abstract, how to prepare figures, and how to give a good presentation. At the end, there was a formal evaluation of SURP, and suggestions were made for modifications in 2016.

On Tuesdays the focus was on paper discussion, led by a graduate student. An academic paper, which would have been read during the week, was discussed for an hour. The purpose of this exercise was to show students how to read academic papers properly, and how to identify important points to look for in order to get as much information out of the papers as possible. Knowing how to read academic papers properly is an important skill to learn, especially for undergraduate students who, prior to this summer, would have been used to reading textbooks instead of academic articles. The styles of writing can be quite different. Another beneficial aspect of the paper discussions was the variety of the papers, which were taken from many different fields within astrophysics every week.

During Thursday meetings, the students' progress on their research was discussed in a scholarly context, allowing students to raise issues about their research and to share any problems that they may have run into. These discussions were extremely useful in giving us practice in presenting our ideas in a scholarly way, but in a friendly environment. These meetings also served as an opportunity for students to discuss any problems they had run into during their research, and for other students to help out. They also allowed students to see what other students were working on during the summer, enabling students to learn more about different fields of research. However, these meetings would have been more beneficial if students were given a longer discussion time. One hour a week seemed far too short to get into much detail with anyone's project, especially when you consider the multiple projects that were discussed in one meeting session. Opening up another one-hour meeting session per week, so that students would meet three times a week, would have been more useful.

Students were also given the opportunity to give two talks on their research in an academic setting, one in the middle of the summer, and one final presentation at the end of the summer. The audience included all of the SURP students as well as many faculty members and graduate students of the department. Students were given an allotted time (12 minutes) in which they had to present their research and answer any questions that the audience might have. Presentations are not always a part of undergraduate classes, which are more often taught in lecture format, so this provided students with a good opportunity to practice their skills giving academic talks. Tips from the "How To Give Good Talks" workshop were extremely useful when having to give the midterm and final presentation. The good thing about having two presentations was that it gave students the opportunity to learn and improve from the first presentation with feedback from their supervisor and to incorporate elements of other students' presentations that worked very well [The presentations were uniformly interesting and professional—JP]. Students were also required to write an abstract of the final presentation for the formal abstract book; a workshop was offered as a guide to how to do this effectively. Having the workshop made writing the abstract a lot easier, and gave students experience with writing abstracts—something that researchers must do very often.

DAA and DI are extremely involved in outreach events. These events provide the general public with an inside look at the University of Toronto and encourage and satisfy interest in astronomy. Outreach is an important part of doing science; it helps raise public understanding about science, whilst also helping to spark interest in the subject that can be particularly helpful to younger generations. SURP students had the opportunity to participate in these outreach events that ranged from astronomy-themed lectures to open-house telescope tours. I was personally able to operate the campus telescopes to engage with an interested audience in order to teach them about different astronomical objects. It was an amazing opportunity to discuss topics that I am extremely passionate about with a different audience, and I will continue to participate in these outreach events after the summer is over. [In 2015–2016, Ariel was hired by DAA as a Teaching Assistant—a rare honor here for an undergraduate—JP].

The Dunlap Institute hosts an annual one-week Instrumentation Summer School at the end of July or beginning of August. The Summer School gives advanced students an introduction to astronomical instrumentation from leading North American experts in the field. SURP students were allowed to attend the Dunlap Summer School lectures. They covered various subtopics within the broader topic of instrumentation, such as Radio and Microwave Astronomy, Fourier Transform Spectrometry, and Adaptive Optics. I found this experience extremely valuable, as the topic of instrumentation is scarcely discussed in an undergraduate classroom, but yet is something that is fundamental to astronomy. I left the lectures with a deeper understanding of the basics of instrumentation that has already helped me in my studies. I found these lectures extremely interesting; they actually sparked a previously unknown interest in radio astronomy, as it is something I knew so little about before attending these lectures.

4. Conclusion

Through this project we were able to apply methods learned to analyze variable star data to determine important information about Mira variable stars and their place in stellar evolution and their structure. We found that, by using light curve and Fourier transforms, bimodal pulsators can be identified, as well as how the Fourier method of analyzing data can result in false and alias periods—something which we found present in literature. By using a select few bimodal stars for further analysis, we were able to plot a Petersen diagram, see Figure 2, in which we found that our method of determining periods accurately matched with theoretical Petersen diagrams. We then used (O–C) diagrams to study the rates of period changes within Mira stars, see Figure 3. We were able to relate the rate of period changes derived for the stars to their current evolutionary state. Finally, combining the knowledge learned about both structural and evolutionary models of Mira variable stars, we attempted to explain the

Random Walk fluctuations observed in the periods of Mira stars. Motivated by the fact that Mira stars contain extremely large convection cells which add a random factor to their properties, we simulated period versus time graphs for a star experiencing random fluctuations. We find that these simulations accurately model the random walk behavior observed from data, see Figure 4.

This project was completed within the Summer Undergraduate Research Program (SURP), which provided a structured format for the research projects done by participating students. The wide range of workshops, lectures, and outreach opportunities offered by SURP perfectly supplemented the work done in this project, and allowed for an enhanced summer experience which provided many critical and necessary skills that are essential for future research. A program such as SURP would be extremely beneficial to any undergraduate student who is interested in pursuing research, and my project could serve as a perfect introductory topic for someone new to research. The topic of variable stars served as an ideal introductory project to research, as important skills were learned and it allowed for the opportunity to build upon the knowledge of stellar astrophysics learned in many introductory courses.

5. Acknowledgements

We thank the AAVSO observers who made the observations on which part of this project is based, the AAVSO staff who archived them and made them publicly available, and the developers of the VSTAR package which was used to analyze them. We also thank Thomas Karlsson for making his excellent Mira-star database publicly available. SURP 2015 was very capably organized by Drs. Nicolas Crouzet, Sebastian Daemgen, Christa van Laerhoven, Laura Newburgh, and Aaron Zimmerman. This project was supported by the University of Toronto Work-Study Program.

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