Studies of the Long Secondary Periods in Pulsating Red Giants

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Abstract We have used systematic, sustained visual observations from the AAVSO International Database and the AAVSO time-series analysis package VSTAR to study the unexplained "long secondary periods" (LSPs) in 27 pulsating red giants. In our sample, the LSPs range from 479 to 2,967 days, and are on average 8.1 ± 1.3 times the excited pulsation period. There is no evidence for more than one LSP in each star. In stars with both the fundamental and first overtone radial period present, LSP is more often about 5 times the former and 10 times the latter. The visual amplitudes of the LSPs are typically 0.1 magnitude and do not correlate with the LSP. The phase curves tend to be sinusoidal, but at least two are sawtooth. The LSPs are stable, within their errors, over the timespan of our data, which is typically 25,000 days. The amplitudes, however, vary by up to a factor of two or more on a time scale of roughly 20–30 LSPs. There is no obvious difference between the behavior of the carbon (C) stars and the normal oxygen (M) stars. Previous multicolor photoelectric observations showed that the LSP color variations are similar to those of the pulsation period, and of the LSPs in the Magellanic Clouds, and not like those of eclipsing stars. Although our results are suggestive of rotational variability, there are several arguments against this. So the cause of the LSPs remains unknown.

1. Introduction

About a third of pulsating red giants show a long secondary period (LSP) an order of magnitude longer than the pulsation period (Wood 2000; Percy and Bakos 2003). LSPs have been known for many decades (O'Connell 1933; Payne-Gaposchkin 1954; Houk 1963) from visual or photographic observations. LSPs have also been discovered photoelectrically in many small-amplitude pulsating red giants (Percy *et al.* 1996, 2001; Percy and Bakos 2003). LSPs were rediscovered in 1999 by Peter Wood and colleagues (Wood *et al.* 1999; Wood 2000) from large-scale survey observations of pulsating red giants in the Large Magellanic Cloud (LMC). Since then, he and others have accumulated many more observations of this phenomenon, and have considered many possible explanations. None of them explains these LSPs satisfactorily. Wood considers this to be the most significant unsolved problem in stellar pulsation.

There are at least three possible mechanisms that are still of interest: the turnover of giant convective cells (Stothers 2010), oscillatory convective modes (Saio *et al.* 2015), or the presence of a dusty cloud, orbiting the red giant, together with a low-mass companion in a close, circular orbit (Soszyński and Udalski 2014), or some combination of these.

Nicholls *et al.* (2009) (hereinafter NWCS) discussed the problem of the LSPs in detail, and "are unable to find a suitable model for the LSPs," so they ended by listing "all the currently known properties of LSPs"—a list of 13 items. A few other properties have been added in the literature since then. The purpose of the present paper is to use the systematic, sustained visual data in the AAVSO International Database (Kafka 2016) to add further "knowns" to this list.

LSPs are not the only unsolved mystery in pulsating red giants. Percy and Abachi (2013) found that the pulsation amplitudes in these stars varied by factors of 2 to over 10 on median time scales of 44 pulsation periods. There are also random cycle-to-cycle period fluctuations in red giants (Eddington and Plakidis 1929; Percy and Colivas 1999) which may be related to large convection cells in these stars.

2. Data and analysis

We used visual observations from the AAVSO International Database (AID; Kafka 2016), and the AAVSO VSTAR time-series analysis package (Benn 2013), which includes both a Fourier analysis and a wavelet analysis routine. Stars (listed in Table 1) with sufficient observations in the AID were chosen for analysis from several sources, including Payne-Gaposhkin (1954), Houk (1963), and Kiss *et al.* (1999), among others. We have included three pulsating red supergiants (SG) for comparison. We have also included V Hya, which has unusually-deep and stable LSP minima and may be an eclipsing binary star (Knapp *et al.* 1999). The columns give the star name, the type (M, C, or SG), the starting JD (otherwise all the AID data were used), the pulsation period P, the LSP, the ratio of these, the mean, maximum, and minimum LSP amplitudes, and notes. The notes are as follows: pcs—sinusoidal phase curve; pcs:—possibly sinusoidal phase curve; pc?—phase curve uncertain, but possibly non-sinusoidal; pcst—phase curve sawtooth; dsp—data sparse; *—see section 3.12. The phase curve is a graph of magnitude versus phase, determined with VSTAR using the known period; it essentially folds all of the observations into one cycle. Note that the mean amplitude was determined by Fourier analysis, the maximum and minimum amplitude by wavelet analysis.

Figure 1 shows the long-term light curve of U Del. The visual observations have also been averaged in bins of 119 days (the pulsation period) to show the LSP more clearly.

3. Results

3.1. Lengths of LSPs

NWCS gave a range of 250 to 1400 days, primarily based on observations of stars in the Magellanic Clouds. Olivier and

Table 1. Pulsation periods and long secondary periods in red giants.

Star	Type	JD(start)	P(d)	LSP	LSP/P	A/Amax/Amin	Notes
TZ And	M	2440093	114.8	1355.1	11.8	0.078 0.096 0.049	pc?, dsp
RZ Ari	M	all	56.5	479.4	8.48	0.043 0.102 0.037	pc?
T Ari	M	2415000	320.6	2617.8	8.17	0.429 0.485 0.315	pc?, *
RX Boo	M	all	160.3	2205.1	13.76	0.108 0.136 0.091	pc?, *
RS Cam	M	2443500	90.5	999	11.04	0.157 0.255 0.082	pc:
U Cam	C	2425400	219.4	2967.4	13.53	0.147 0.151 0.147	pcs
IX Car	SG	2442500	371.5	4608.3	12.40	0.188 0.249 0.213	pcs:, dsp
AA Cas	M	2437500	80.1	866.8	10.82	0.041 0.066 0.022	pc?
SS Cep	M	2432500	101.1	955.2	9.45	0.083 0.137 0.064	pcs:
RT Cnc	M	all	89.3	691.7	7.75	0.067 0.116 0.035	pc?
FS Com	M	2440000	55.7	688.7	12.36	0.033 0.080 0.042	pc?
Y CVn	C	2430000	160	2008.9	6.88	0.071 0.096 0.049	pcs, *
AW Cyg	C	2431500	209:	2289:	6.40	0.084 0.091 0.081	pcs:, *
BC Cyg	SG	2437500	698.8	3459.6	4.95	0.128 0.171 0.010	pc?
U Del	M	all	119.0	1162.8	9.77	0.211 0.257 0.153	pcs
RY Dra	C	2432500	276.7:	1135.6:	4.10	0.108 0.137 0.064	pcs, *
TX Dra	M	2431702	77.5	711.8	9.18	0.080 0.193 0.101	pcs:, dsp
RW Eri	M	all	91.4	952.0	10.42	0.148 0.172 0.126	pcs:
Z Eri	M	2430000	78.5	692.4	8.82	0.065 0.197 0.080	pc?
TU Gem	C	all	214.6	2413.7	11.25	0.093 0.101 0.097	pcs:
g Her	M	2430000	87.6	878.7	10.03	0.164 0.222 0.132	pcs, *
X Her	M	2430240	176.6	1185.3	6.71	0.060 0.134 0.047	pc?
V Hya	C	all	531.4	6907.4	13.00	1.132 1.160 1.092	*
RV Lac	M	all	70.0	632.6	9.04	0.320 0.453 0.261	pcst
Y Lyn	M	all	134.7	1258.7	9.34	0.322 0.398 0.149	pcst
BQ Ori	M	2430000	246.6	2147.8	8.71	0.034 0.047 0.026	pc?
W Ori	C	2430000	210.7	2335.1	11.08	0.197 0.213 0.176	pc?
SU Per	SG	2432500	469.0	3355.7	7.16	0.115 0.122 0.111	pc?
UZ Per	M	all	186.3	893.4	4.80	0.293 0.357 0.236	pc?
τ4 Ser	M	all	111	1151.2	13.51	0.104 0.117 0.071	pcsa, *:
ST UMa	M	2430000	90.3	623.1	6.90	0.070 0.117 0.041	pc?
V UMi	M	2430644	72.9	757.3	18.7	0.078 0.103 0.031	pcs:, *

Wood (2003) analyzed 26 Galactic stars with LSPs between 500 and 3,735 days (excluding V Hya, which is peculiar and possibly a binary). Our study includes both short-period and longer-period stars in the Milky Way. The Galactic stars in Table 1 may have different properties than the low-metallicity stars in the Magellanic Clouds studied by Wood and others. Excluding supergiants, our range is 479 to 2,967 days (Table 1; Figure 2). There may be stars with shorter or longer LSPs, presumably with pulsation periods shorter or longer than those in Table 1.

3.2. LSPs in red supergiants

It is important to note that LSPs are also found in pulsating red *supergiants* (Kiss *et al.* 2006; Percy and Sato 2009); the LSP phenomenon seems to be continuous from class III to class II to class I stars. Kiss *et al.* (2006) used Fourier techniques to analyze visual measurements; Percy and Sato (2009) used self-correlation techniques to analyze similar datasets. There are about ten stars for which the above-mentioned papers obtained consistent pulsation periods and LSPs; for these, the median value of LSP/P is 6.0 and the median LSP visual amplitude is 0.10 magnitude, similar to those in red giants. For red supergiants in the Large Magellanic Cloud, the median LSP/P is about 4, and the median V amplitude is about 0.08 (Yang and Jiang 2012). Kiss *et al.* (2006) concluded, on the basis of the Lorentzian envelopes of the peaks in the Fourier spectra, and the strong 1/f noise, that large convection cells play an important role in the behavior of these red supergiant stars.

3.3. The amplitudes of the LSPs

The median LSP amplitude for the stars in Table 1 is 0.10 for both the M stars and the C stars. There is no evidence that it varies with LSP (and therefore with the size and/or temperature of the star), according to Figure 3, which shows the LSP amplitude as a function of LSP. Note that we find LSPs in short-period pulsators such as RZ Ari, whose variability is best studied photoelectrically (Percy *et al.* 2008). Nicholls *et al.* (2010) show that the LSP *velocity* amplitude is constant, at a few km/s, over a very wide range of LSPs.

It is said that Mira stars do not have LSPs, but low-amplitude LSPs may be hidden by the large-amplitude pulsation, and by the complex Fourier spectra of these stars, which arise from the systematics in the time distribution of the observations, and from the stars' random changes in period and amplitude. For instance, the Fourier spectra of AAVSO visual data on Mira stars tend to have strong one-cycle-per-year aliases at periods of a few thousand days (the possible values of the LSPs), and the noise level in the spectra exceeds 0.1 magnitude.

3.4. The phase curves of the LSPs

For most of the stars in our sample, the LSP phase curves are indistinguishable from sine curves, with the exception of a very small number which have sawtooth or possibly sawtooth shapes. The clearest example is Y Lyn (Figure 4). This figure shows the long-term light curve of Y Lyn; the visual observations have also been averaged in bins of 134.7 days (the pulsation period) to show the sawtooth shape of the LSP light curve more clearly. The other star with a distinctly saw-tooth phase curve is RV Lac. We note that both these stars have higher-than-average LSP amplitudes. The model of Soszyński and Udalski (2014) predicts a different shape for the phase curve; see section 4.

3.5. Is the LSP always ten times the fundamental pulsation Period?

Some stars with LSPs pulsate in the fundamental mode, and others in the first overtone. Is the LSP always ten times the excited period, no matter which mode is excited?

Fuentes-Morales and Vogt (2014) used the ASAS database to study 72 pulsating red giants. Of the stars which they identified as triply periodic (their Table 2), eight appear to have an LSP and a fundamental (P0) and a first overtone (P1) radial period present. In three of these, LSP/P1 is closer to 8.1 than LSP/P0; in two of these, LSP/P0 is closer, and in three of them, both ratios are equidistant from 8.1. In Kiss *et al.* (1999)'s list of triply-periodic stars, there are nine with LSP, P0, and P1. In six cases, LSP/P1 is closer to 8.1, in two, LSP/P0 is closer, and in one case, they are equidistant from 8.1. In our sample, there are four stars (RX Boo, RS Cam, TX Dra, and X Her) which appear to have these three periods present. In three, LSP/P0 is closer to 8.1; in the other, LSP/P1 is closer to 8.1. We conclude that, although LSP is slightly more often about 8.1 times the first overtone radial period, this is not always the case. This may explain some of the scatter in Figure 2.

A different approach is to consider a plot (Figure 5) of LSP/P0 and LSP/P1 versus LSP. For stars with LSP less than about 1,500 days, LSP/P0 is about 5–6, LSP/P1 about 9–11. This suggests that the LSP is generally about 5–6 times the fundamental period. For stars with LSP greater than 1,500 days, the behavior is much less consistent.

3.6. Uniqueness of the LSP

Some theories of the LSP suggest that there may be two or more long timescales in a pulsating red giant, such as the turnover time of a bright or dark convective cell, and the rotation period of the star. There are already four timescales which we have identified in our sample: (1) the pulsation period; (2) the LSP; (3) the timescale for increase and decrease of the amplitude of the pulsation period; and (4) the timescale for increase and decrease of the amplitude of the LSP. As noted below, timescale (3) is not the same as the LSP.

We have used Fourier analysis to look for stars in our sample which might have more than one LSP, but there are none in which a second LSP is significantly above the noise level. For example, the LSP light curves in Figures 1 and 4 appear to be monoperiodic.

3.7. LSP Period stability

For most of the stars in Table 1—those for which the observations are both dense and sustained—the half-width at half-maximum errors in determining the LSP are 3 to 5 percent. Figure 6 shows the Fourier spectrum of TZ And, showing how the HWHM uncertainly of the LSP was determined. We adopt 3 percent as a reasonable intrinsic uncertainty caused by the finite length of the dataset and the scatter in the visual observations. We then used wavelet analysis to study changes in the LSP with time. We did not find any stars for which LSP varied by more than three times the error—that is, by 10 percent or more, over the timespan of the observations, which is typically 25,000 days. A possible exception may be Z Eri, for which the mean LSP is 722 days, the range is 695 to 740 days, and the formal error of the LSP is only 7 days. This error seems unreasonably low, however, being only 1 percent.

3.8. LSP amplitude stability

Percy and Abachi (2013) showed that, for a small sample of red giants with LSPs, the amplitudes of most of the LSPs varied by a factor of up to 2, on a time scale of typically 30 LSPs or greater. Figure 7 shows the amplitude of the LSP of RT Cnc as a function of time. The amplitude rises and falls on a time scale of about 20 LSPs. This is true of the other stars in Table 1, as it was for the LSPs of the stars studued by Percy and Abachi (2013). It is difficult to determine the time scale of the LSP amplitude variability because of its length, but one way of doing so is shown in Figure 8, which plots a measure of the *change* in LSP amplitude against the LSP. Stars with longer LSPs change less in amplitude, presumably because, for these stars, the time scale of amplitude variability, being tens of times greater than the LSP itself, is longer than the timespan of our observations. This interpretation of Figure 8 is therefore consistent with the results of Percy and Abachi (2013).

3.9. Correlation of pulsation amplitude with LSP Phase

NWCS note that the primary pulsation is visible in the light curve at all times throughout the LSP and the primary *period* does not significantly change with LSP phase. Because the cycles of increase and decrease of the primary pulsation period's *amplitude* vary on a median time scale of 44 periods (Percy and Abachi 2013), whereas the LSP is about 8.1 times the excited pulsation period, it follows that the pulsation period's *amplitude* is not correlated with the LSP phase. For instance: for W Ori, the cycles of pulsation amplitude increase and decrease are about 4,500 days, whereas the LSP is about half that.

3.10. Oxygen (M) stars versus carbon (C) stars

In our previous studies of red giants, Percy and Yook (2014) identified one possible difference in pulsational behavior between oxygen (M) stars and carbon (C) stars, namely that, in C stars, amplitude changes correlated with period changes, whereas this was not true for M stars. We have therefore looked in our database to see whether there are any differences between M and C stars in the LSPs or their amplitudes. In doing this, we have been careful to compare stars with similar periods. Carbon stars have longer pulsation periods, on average, than oxygen stars, because the carbon-star phenomenon is associated with a more advanced stage of evolution, when the star is larger and cooler. Figures 2 and 8 show no obvious differences between the behavior of the two classes of star.

3.11. LSP color to light variations

Derekas *et al.* (2006) pointed out that, on the basis of their observations, the relative color amplitudes were similar, for the LSPs, to those for the pulsation period, but this was not true for stars with ellipsoidal variability. Percy *et al.* (2008) used merged AAVSO and robotic telescope photometry to show that the same was true for the LSPs of 12 small-amplitude pulsating red giants (EG And, RZ Ari, ψ Aur, BC CMi, TU CVn, FS Com, α Her, V642 Her, 30 Her, Y Lyn, UX Lyn, TV Psc), with the possible exception of EG And, which appears to be a genuine symbiotic spectroscopic binary (their Figures 7 and 8). Specifically, these stars had a visual LSP amplitude about twice the red amplitude which, in turn, was about twice the near-infrared amplitude; they are brighter when hotter. These relative amplitudes are slightly larger, on average, than the $\Delta V/\Delta I$ ratios shown in Figure 4 of Soszyński and Udalski (2014), but are within the same range. Also, the metallicities of the Galactic and LMC stars are quite different.

3.12. Notes on individual stars

T Ari This star, because of its large amplitude, is classified as a Mira star.

RX Boo There may also be a 372.99-day period.

Y CVn The pulsation period is uncertain, but is most likely 160 days.

AW Cyg The pulsation period (most likely 358 days) and the LSP are uncertain.

RY Dra The 276.7-day pulsation period is consistent with the DIRBE photometry (Price *et al.* 2010), and the 1,135.6-day LSP in the AAVSO data seems secure. But the DIRBE data suggest that the LSP may be longer.

g Her There may also be a 61.21-day pulsation period.

V Hya This is a peculiar pulsating red giant with an LSP (6,907 days) which has a large amplitude (1.13 magnitudes), a V-shaped phase curve, and a stable LSP and LSP amplitude—very much like the RVB star U Mon. Knapp *et al.* (1999) suggest that this is a binary in which the secondary component is surrounded by dust.

τ4 Ser The pulsation period is uncertain, but is most likely 111 days.

V UMi There may also be a 125.45-day pulsation period.

4. Discussion

Numerous possible explanations for the LSPs have been proposed and examined but, as NWCS have described, most have not been successful. These include radial or non-radial pulsation—unless there is some non-standard pulsation mode yet to be discovered. Standard binary models fail to explain the highly non-random distribution of velocity-curve amplitudes, shapes, and orientations (but the situation is reminiscent of the "Barr effect" (Percy 2015) in which the velocity curves of spectroscopic binary stars were distorted by mass transfer).

Saio *et al.* (2015) have proposed that oscillatory convective modes are a possible explanation for the LSPs. The timescales are consistent with the LSPs, assuming that the modes are actually excited. The amplitudes, shapes, and color dependences of the phase curves are in reasonable agreement with observations. We would need some explanation for the variable amplitudes of the LSPs. Saio *et al.* (2015) state that there is a minimum luminosity for the presence of oscillatory convective modes. We find LSPs in many less-luminous stars, with amplitudes comparable to those in more luminous ones.

Another interesting and still-viable explanation is the rise and fall of giant convective cells, which are known to occur in cool stars (Stothers 2010). The time scales of rise and fall are similar to those of LSPs, but we must remember that these stars are also rotating, so this phenomenon would be modulated by the rotation period. We do not find evidence for two different LSPs in the same star (section 3.6). Also: we would need an explanation for the variable amplitude of the LSPs.

There are possible mechanisms which are based on rotational modulation. These include the effect of a dusty cloud, orbiting the red giant with a low-mass companion in a close, circular orbit (Soszyński and Udalski 2014); the period would be the orbital period but, since the authors suggest that the red giant would be co-rotating with the companion, the period would be essentially the rotation period. It may seem improbable that a third of red giants should have such configurations, but we should remember that the not-uncommon LSPs in RV Tauri stars (the so-called RVb stars) are thought to be due to the effects of a binary companion (Percy 1993; Waelkens and Waters 1993). Another rotation explanation would be the presence of a large, hot/bright or cool/dark convective cell with a lifetime of a few LSPs.

There are problems with any rotational explanation. Olivier and Wood (2003) carried out a careful spectroscopic study of 26 red giants with LSPs and 30 without. The power spectra of the flux spectra were analyzed for rotation and microturbulence according to the methods of Gray (1976). There was no significant difference between the two samples so, for instance, there was no evidence that the LSP stars were seen equator-on and the non-LSP stars were seen pole-on, or that high or low rotation was related to the cause. Olivier and Wood (2003) derived a general *upper limit* to the equatorial rotational velocity of 3 km/s. This would lead to LSPs in excess of about 3,000 days, whereas most LSPs are much less than this.

Furthermore: theoretical models of evolution, with rotation, predict very slow rotation in red giants, unless some process could transfer angular momentum to the outer layers (Privitera *et al.* 2016 and references therein). Rotation could be spun up by the engulfment of Jupiter-mass planets (for example, Privitera *et al.* 2016), but this would result in a distribution of rotation periods. In fact, the natural distribution of initial rotation rates in red giant progenitors should also produce a distribution of rotation periods in the red giants, whereas the LSPs are well

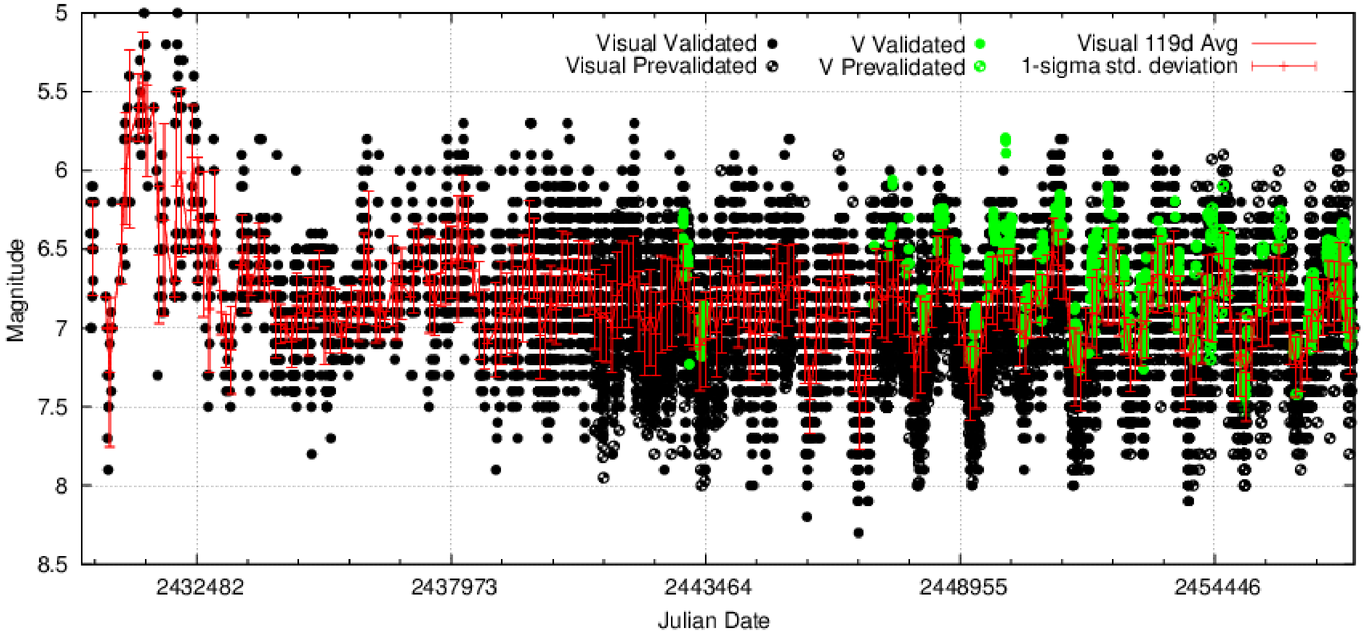

Figure 1. The long-term AAVSO light curve of U Del, also showing the data averaged in bins of 119 days (the pulsation period) to shown the LSP light curve (red curve, points with error bars) more clearly. The LSP, as determined by VSTAR, is 1,162.8 days, and this is consistent with the value which would be determined by counting cycles in this figure. We have plotted all the observations, to show how the density of observations and the clarity of the LSP increase after JD 2440000. The black points are visual; the green ones (lighter circles) are photoelectric V.

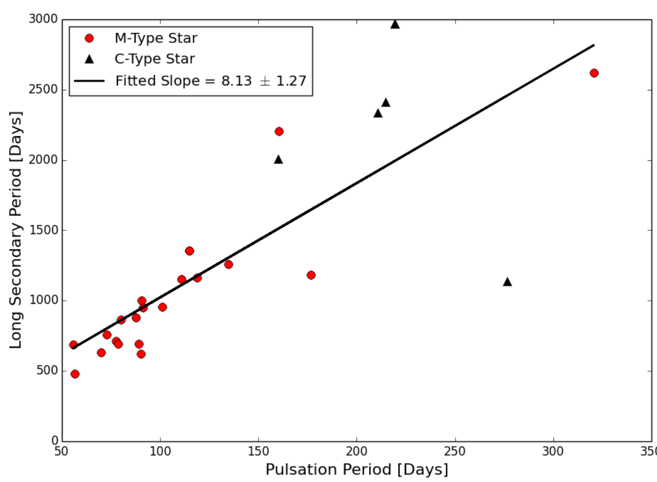

Figure 2. The LSP as a function of pulsation period. The slope of a linear fit is 8.1 ± 1.3.

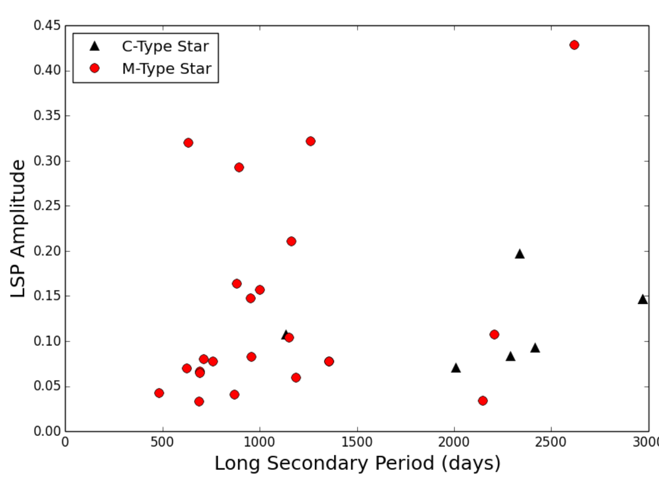

Figure 3. The LSP amplitude, in magnitudes, as a function of LSP. There is much scatter, but the median amplitude is about 0.10 magnitude, and there is no obvious correlation between the two.

correlated with the pulsation period and therefore with the radius of the star.

On the other hand, Zamanov *et al.* (2008) have measured the equatorial rotational velocities of 28 M-type red giants, and found rotational velocities closer to 5 km/s. These would result in shorter rotation periods. These authors used different methods of analysis—cross-correlation, and absorption-line width—to estimate the velocities. These methods may well under-estimate the contribution of convection and velocity gradients due to winds and pulsation on the spectral line profiles, because the calibration and cross-correlation were done against K-type (rather than M-type) comparisons, so their rotational velocities are upper limits. In summary: rotational mechanisms for the LSP phenomenon are problematic, but not entirely ruled out.

5. Conclusions

We have used systematic, sustained visual observations from the AAVSO International Database and the AAVSO time-series analysis software package VSTAR to study the LSPs in 27 red giants. The LSPs are, on average, 8.1 ± 1.3 times the excited pulsation period, and the median visual amplitude is 0.1, independent of the LSP. The LSPs are stable over many decades, but the amplitudes rise and fall on a time scale of a few tens of LSPs. The LSP phase curves are not well-defined, but at least two appear to have a sawtooth shape. The similarity between the LSP and the rotation period of the star suggests that the LSP phenomenon may be due to rotational variability, modulated by a large bright or dark convective cell, or by obscuring material near the photosphere, but there are also strong arguments against the rotational hypothesis, as explained above.

We also note that visual observations, despite their lower accuracy, are able to delineate the long-term behavior of these stars because they are sustained over up to a century. Whether

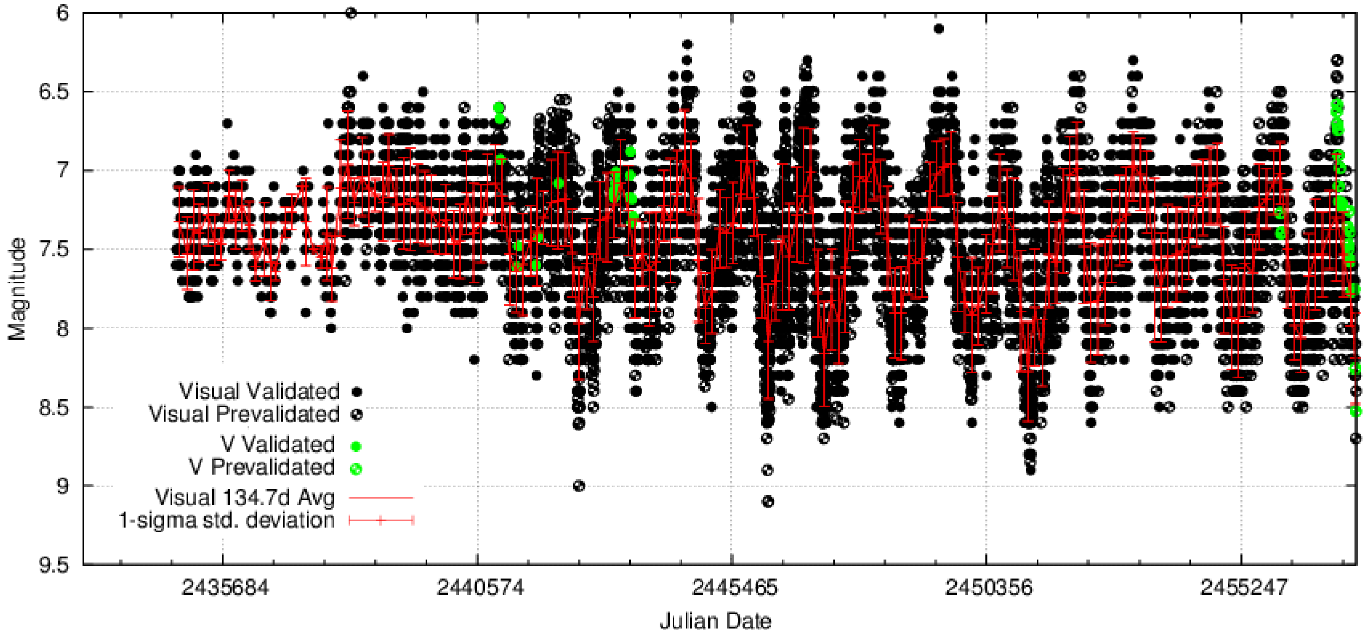

Figure 4. The long-term light curve of Y Lyn, also showing the data averaged in bins of 134.7 days (the pulsation period) to show the sawtoothed shape of the LSP light curve (red curve, with error bars) more clearly. The LSP, as determined from vstar, is 1,258.7 days, and this is consistent with the value which would be obtained by counting cycles in this figure. We have plotted all the observations, to show how the density of the observations and the clarity of the LSP increase after JD 2440000. The black points are visual; the green ones (lighter circles) are photoelectric V.

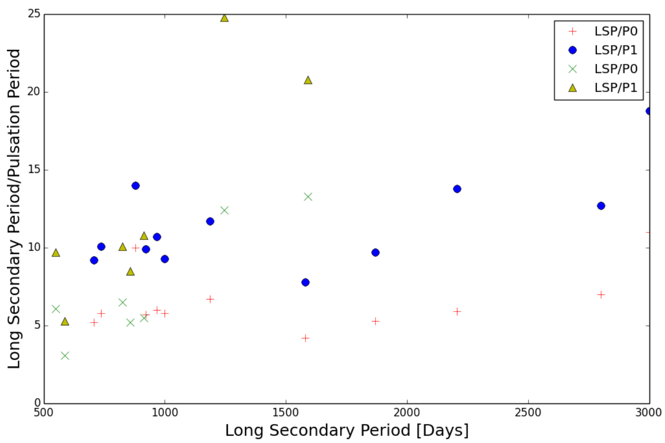

Figure 5. For stars with both P0 and P1 excited: the ratio of the LSP to P0 (plus signs and x's), and to P1 (filled circles and triangles). The plus signs and filled circles are from the visual data of Kiss *et al.* (1999) and from Percy and Huang (2015); the x's and filled triangles are from the ASAS data of Fuentes-Morales and Vogt (2014).

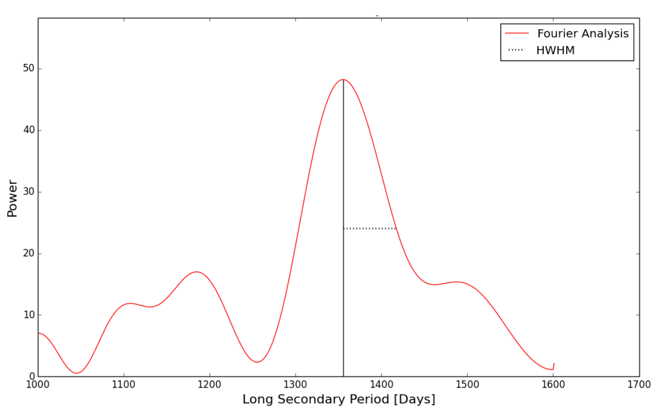

Figure 6. The Fourier spectrum of TZ And, showing how the uncertainty σ in the LSP is determined from the half-width at half-maximum of the LSP peak.

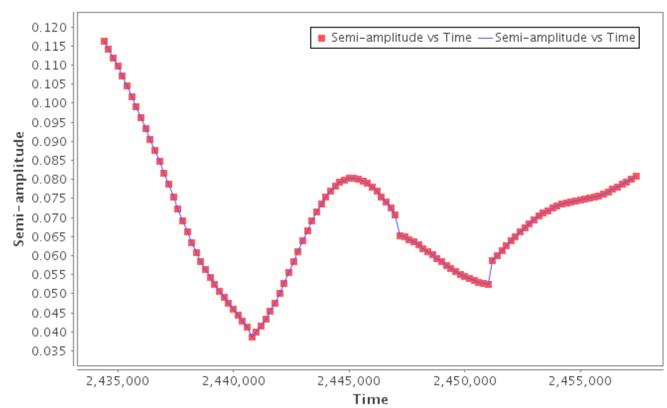

Figure 7. The LSP amplitude of RT Cnc as a function of time, determined by wavelet analysis. The amplitude rises and falls on a time scale of about 20 LSPs.

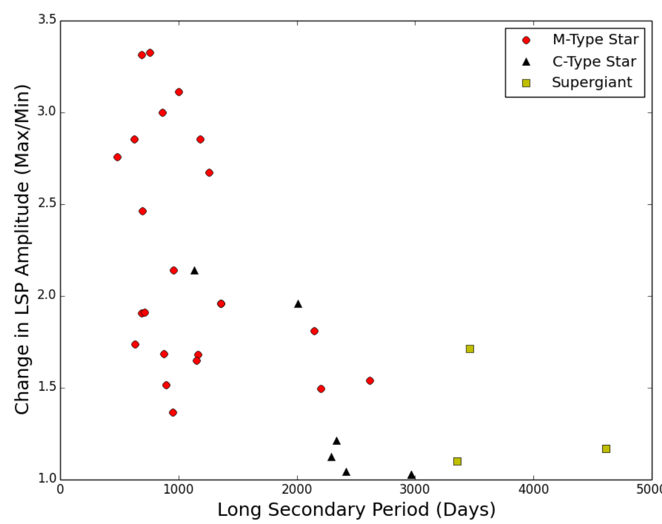

Figure 8. The relative change in LSP amplitude (max/min) over the timespan of the data, as a function of LSP. As explained in the text, this suggests that the LSP amplitude varies slowly, on a time scale of a few tens of LSPs.

or not the rotational hypothesis is still viable, we believe that we have added a few more "knowns" to the NWCS list, and have brought us a bit closer to solving this longstanding mystery.

6. Acknowledgements

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