Pulsation Properties of Carbon and Oxygen Red Giants

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Abstract We have used up to 12 decades of AAVSO visual observations, and the AAVSO VSTAR software package to determine new and/or improved periods of 5 pulsating biperiodic carbon (C-type) red giants, and 12 pulsating biperiodic oxygen (M-type) red giants. We have also determined improved periods for 43 additional C-type red giants, in part to search for more biperiodic C-type stars, and also for 46 M-type red giants. For a small sample of the biperiodic C-type and M-type stars, we have used wavelet analysis to determine the time scales of the cycles of amplitude increase and decrease. The C-type and M-type stars do not differ significantly in their period ratios (first overtone to fundamental). There is a marginal difference in the lengths of their amplitude cycles. The most important result of this study is that, because of the semiregularity of these stars, and the presence of alias, harmonic, and spurious periods, the periods which we and others derive for these stars—especially the smaller-amplitude ones—must be determined and interpreted with great care and caution. For instance: spurious periods of a year can produce an apparent excess of stars, at that period, in the period distribution.

1. Introduction

Red giants are unstable to radial pulsation. In general: the larger, cooler, and more luminous the star, the longer the period, and the greater the amplitude. Smaller red giants tend to pulsate in higher overtones, larger ones in lower overtones—the fundamental or first overtone. A few stars pulsate in two (or more) modes, and this is particularly useful since it provides twice as much precise information.

Percy and Abachi (2013) showed that the amplitudes of almost all pulsating red giants change significantly—by factors of up to 10—on time scales of a few tens of pulsation periods. Percy and Khatu (2014) showed that the same was true of pulsating red supergiants, and Percy and Kim (2014) found the same behavior in many pulsating yellow giants and supergiants. Percy and Yook (2014) investigated whether these changes in amplitude resulted in small changes in period, due to non-linear effects in the pulsation. Indeed, they did—*but only in carbon (C-type) red giants*. That led us, in the present study, to investigate whether there were any other differences in the pulsation properties of C red giants and normal oxygen (M) red giants. Since period ratios are a sensitive probe of stellar structure, we made a special effort to find biperiodic C-type stars, and to check the periods of "known" biperiodic C stars. In the course of doing this, we examined the periods and amplitudes of many C red giants, and were able to refine these and, in some cases, identify literature periods which were actually alias, harmonic, or spurious periods (see section 2). We also analyzed some M red giants, to check and refine their periods. Almost all of the stars in our sample had ranges in visual magnitude which were less than 2.5, so they would be classified as semiregular (SR) rather than Mira. See Mattei *et al.* (1997) for a discussion of the classification of red variables. To emphasize: our initial aim was to look for possible differences between the pulsational behavior of C and M stars. This led to the second aim: to look more critically at the periods which we and others derived for these stars.

Multiperiodicity provides one very precise way of analyzing pulsating stars, and looking for differences between subgroups of those stars—such as between C and M red giants—and comparing observed periods with those predicted by theory. Petersen (1973) was a pioneer in such work. He plotted the ratio of periods, such as the ratio of the first overtone period to the fundamental period (P1/P0), against the fundamental period (P0). Such a plot is now called a Petersen diagram.

The most extensive study of multiperiodicity in pulsating red giants is by Kiss *et al.* (1999), hereafter KSCM; see also Mattei *et al.* (1997). We reanalyzed many of their stars, especially those which were C stars, and a selection of M stars for comparison with the C stars. Our database is, of course, up to 15 years longer than theirs.

Many pulsating red giants also have a "long secondary period" (LSP) of unknown origin. Based on our previous studies, and especially on massive photometric surveys such as MACHO and OGLE, these LSPs are known to be about 10 times the fundamental pulsation period. In the present study, we are not immediately concerned with these.

2. Data and analysis

We used visual observations, and for a few stars V observations, from the AAVSO International Database (AID; Henden 2014, Kafka 2015), of the stars listed in the tables. For determination of the average period and amplitude of the star during the time of the observations, we used the Fourier routine (DC-DFT) in the AAVSO software package VSTAR (Benn 2013). Our approach was somewhat different than that of KSCM, who used and merged three different sources of visual data, and first binned them in 10-day bins.

The red giants in our sample generally have pulsation periods in the range of 100–500 days. Because of seasonal gaps in the data, they can show *alias* periods which differ in frequency from the true periods by N cycles per year, where N is usually ±1, but are lower in amplitude. They are identified by the fact that they differ in frequency from the strongest frequency by ±N/365.25 cycles per day, where N is usually 1.

Because of a physiological effect called the Ceraski effect (Ceraski was an eminent Russian photometrist of a century ago; see Sharonov (1933) for an experimental investigation of this effect), visual observations can also have *spurious* periods of about one year. Although the Ceraski effect is generally thought to be physiological in nature, it is possible that some or all of it may be due to aliasing of very long-term, low-frequency variability in the star (Percy 2015, in preparation). Any period of about one year is therefore suspect. These spurious periods generally have amplitudes less than 0.1 magnitude. Both alias and spurious periods are a special problem for stars, such as red giants, which have one or more true periods of a few hundred days. They are even more of a problem if the star is multiperiodic. There is an additional problem: if the light curve is non-sinusoidal, there are harmonic periods in the Fourier spectrum and, in the case of fundamental-mode pulsators, the P/2 harmonic is very close to possible first-overtone periods which are 0.4–0.6 times the fundamental period. Harmonic periods are not true periods; they are a mathematical result of representing a non-sinusoidal light curve by a sum of sinusoids.

Figure 1 shows the DC-DFT power spectrum for V Boo, a simple monoperiodic pulsator. The 258-day period is flanked by two alias periods with much lower amplitude. Figure 2 shows the DC-DFT power spectrum for RR Her, an example of a biperiodic pulsator. The 237-day period is flanked by two alias periods, but there is an additional period of 124 days which we assume to be a real first-overtone period.

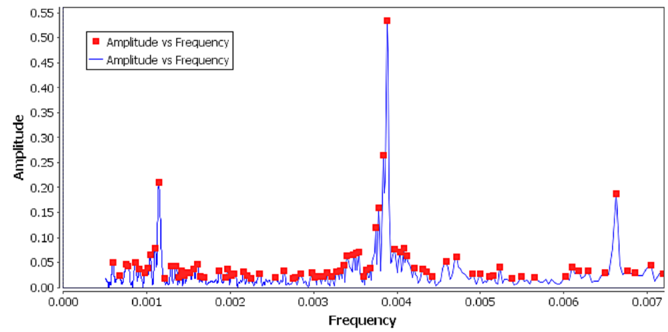

Figure 1. The DC-DFT power spectrum of V Boo, an example of a monoperiodic pulsator. The 258-day period is flanked by two alias periods, whose frequencies differ from the true frequency by ±1 cycle per year. In this and Figure 2, the red symbols are the "top hits"—the peaks of the spectrum. The continuous line is the spectrum itself.

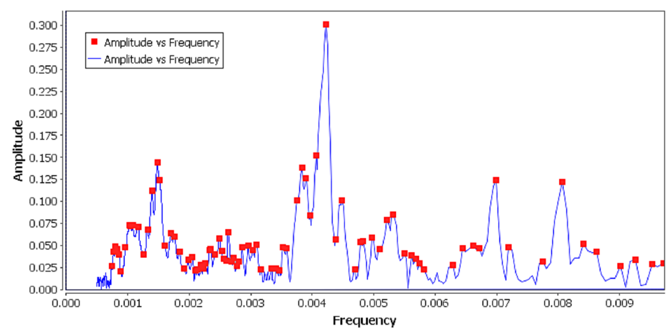

Figure 2. The DC-DFT power spectrum of RR Her, an example of a biperiodic pulsator. The 237-day period is flanked by two alias periods, but there is an additional period of 124 days which we assume to be a true period.

3. Results

3.1. Double-mode pulsators

Table 1 lists the results for five C red giants which we believe to have two radial periods which are not alias, spurious, or harmonic periods, and a selection of twelve M red giants which are also biperiodic, for comparison. The columns give the name of the star, the spectral type (from SIMBAD), the periods in days, the amplitudes, the ratio P1/P0, the dominant period DP (P0 or P1), and notes about each star. The uncertainties in the periods, as estimated from the half-width at half-height in the Fourier spectrum, are 1–2 percent, unless otherwise indicated in the notes. The amplitudes in Tables 1, 2, and 3 are average

Table 1. Double-mode Carbon and Oxygen Red Giants.

Star	*SpT*	*P1(d)*	*A1*	*P0(d)*	*A0*	*P1/P0*	*DP*	*Notes**
RU And	M5-6e	146.8:	0.16	234.3	0.29	0.532	P0	3
SV Cas	M6.5	239.9	0.32	455.6	0.56	0.498	P0	1,3,8
RU Cyg	M6-8e	233.9	0.31	443.0	0.14	0.527	P1	1,3,8
RZ Cyg	M7.0-8.2e	275.9	0.72	537.6	0.54	0.513	—	1,3,8
AH Dra	M7	105.9	0.18	190.0	0.26	0.557	P0	1,3,8
AY Dra	M7:	130.5:	1.51	262.8	2.83	0.497	P0	3,6
BQ Ori	M5-8IIIe	125.8:	0.10	246.5	0.14	0.510	P0	1,3,8
DP Ori	M6.5	146.9	0.12	245.8	0.23	0.515	P0	—
W Tau	M4-6.5	127.5	0.09	240.8	0.20	0.529	P0	1,3,3,7
Z UMa	M5III	98.8	0.14	189.2	0.30	0.522	P0	1,3,8
EP Vel	M6	258.9	0.57	513.6	0.55	0.504	P1	—
RU Vul	M3-4e	154.7	0.18	368.2	0.28	0.420	P0	1,3,8
S Cep	C7,4e (N8e)	208.3	0.20	486.1	0.95	0.496	P0	3
V Cyg	C5,3-7.4e (Npe)	195.6	0.33	420.9	1.43	0.497	P0	3,6
RS Cyg	C8,2e (N0pe)	195.1	0.17	419.0	0.49	0.504	P0	3
RR Her	C5,7-8,1e (N0e)	124.2	0.12	236.7	0.31	0.525	P0	3,5
SY Per	C6,4e (N3e)	231.5	0.18	479.2	0.82	0.483	P0	3

**The amplitudes are average values over the dataset. Also see notes to individual stars in section 3.5. (1) also analyzed by KSCM; (2) low amplitude; (3) see note in section 3.5; (4) extremely low amplitude (less than 0.10 mag.); (5) there is a gap in the data; (6) the data are sparse; (7) monoperiodic according to KSCM; (8) biperiodic according to KSCM (these periods refer to pulsation periods, not LSPs).*

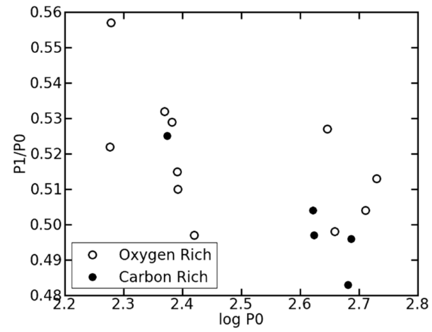

Figure 3. The Petersen diagram for the biperiodic stars in Table 1; it plots the ratio of P1/P0 against logP0. The filled circles are carbon stars; the open circles are normal (oxygen) stars.

values over the dataset; the amplitudes of pulsating red giants vary significantly on timescales of dozens of pulsation periods (Percy and Abachi 2013).

Figure 3 shows the Petersen diagram for the stars in Table 1. It plots P1/P0 against the logarithm of P0, in days. The filled circles are C stars; the open circles are M stars. For longer periods, the period ratios of the C stars are marginally lower than those of the M stars, but not significantly so. The trend and levels in this diagram are consistent with the theoretical models of Xiong and Deng (2007), though the observed period ratios are slightly larger than the theoretical ones for the longer-period stars. There is some scatter in the graph. This may be observational in nature, or it may be due to actual differences between the stars. For instance: Wood (2015) has presented convincing evidence for mass differences among pulsating red giants in the Large Magellanic Cloud.

3.2. Carbon-rich stars

Table 2 lists the results for monoperiodic C red giants. The columns give the name of the star, the spectral type (from SIMBAD), the period in days, the average amplitude, and notes about the star. Tables 2 and 3 include a few stars which we studied but were unable to find periods for.

3.3. Oxygen-rich stars

Table 3 lists the results for some monoperiodic M red giants. The columns give the name of the star, the spectral type (from SIMBAD), the period in days, the average amplitude, and notes about each star.

3.4. Amplitude variations

Because of our recent studies of the pulsational amplitude variations in red giants (e.g. Percy and Abachi 2013), we used the wavelet (WWZ) routine in VSTAR to study the cyclic variations in the pulsation amplitudes of some of the biperiodic stars in Table 1, using the same methodology as Percy and Abachi (2013). We also analyzed GY Aql, W Ori, and RZ Peg, which were initially thought to be biperiodic. We determined the number of cycles of amplitude increase and decrease N and the average length L of the cycles, and expressed them in units of the pulsation period P. The results are given in Table 4; the first few stars are M type, and the last few stars are C type. The columns give, for the fundamental and first overtone mode, the period P, the number of amplitude cycles N, and the ratio of L to P, and the initial JD. For the biperiodic M stars, we obtained L/P = 43 ± 16 (standard deviation) or ±4 (standard error of the mean). For the biperiodic C stars, we obtained L/P = 31 ± 18 (standard deviation) or ±6 (standard error of the mean). The median values were 42 and 32, respectively. The difference between the two values of L/P is suggestive but not statistically significant. See Percy and Abachi (2013) for many examples of amplitude-versus-time graphs, and for a discussion of the determination of L, and the uncertainties therein.

Table 2. Periods and amplitudes of monoperiodic Carbon Red Giants.

Star	*SpT*	*P(d)*	*A*	*Notes**
ST And	C4,3-6,4e	338.2	0.50	1,3,8
VX And	C4,5J(N7)	367.0	0.29	1,3,6,8
VY And	C3,5J-4,4-5(R8)	445.1:	0.13	2
AQ And	C5,4(Nb)	336.1	0.13	1,2,3,8
BI And	S8,8	—	—	3,6
V Aqr	M6e	241.2	0.22	1,3,7
V Aql	C5,4-6,4(N6)	381.9:	0.09	1,3,4,6,8
S Aur	C4-5,4-5(N3)	367.7:	0.61	1,3,7
V Aur	C6,2e(N3e)	352.6	1.24	—
S Cam	C7,3(R8e)	327.7	0.79	1,3,7
U Cam	C3,9-6,4e(N5)	219.4	0.08	1,3,4,5,8
ST Cam	C5,4(N5)	365.6:	0.11	1,2,3,6,8
T Cnc	C3,8-5,5(R6-N6)	486.8	0.35	1,5,7
X Cnc	C5,4(N3)	370.7:	0.08	1,3,4,8
Y CVn	C5,4J(N3)	358.2:	0.06	1,3,4,8
RT Cap	C6,4(N3)	407.9	0.25	1,3,5,6,7
WZ Cas	C9,2JLi(N1p)	370.8	0.15	1,2,3,8
V CrB	C6,2e(N2e)	357.6	1.49	—
U Cyg	C7,2-9,2e(Npe)	465.3	1.25	—
RV Cyg	C6,4e(N5)	362.6:	0.13	2,3
SV Cyg	C5,5-7,4(N3)	369.9:	0.09	3,4,5
TT Cyg	C5,4e(N3e)	373.1:	0.03	1,3,4,5,8
WX Cyg	C8,2JLi(N3e)	409.4	1.12	—
AW Cyg	C4,5(N3)	357.5:	0.07	1,3,4,8
V460 Cyg	C6,4(N)	164.8	0.06	1,3,4,7
T Dra	C6,2-8,3e(N0e)	422.3	1.27	—
RY Dra	C4,5J(N4p)8	276.8:	0.06	1,3,4,7
UX Dra	C7,3(N0)	364.1:	0.06	1,3,4,8
R For	C4,3e(Ne)	388.5	1.24	3
U Hya	C6.5,3(N2)(Tc)	369.8:	0.08	1,3,4,7
V Hya	C6,3-7,5e(N6e)	—	—	1,3,7
Y Hya	C5,4(N3p)	363.3:	0.14	2,3,6
RY Hya	C6,4e(Nb)	—	—	3
T Lyr	C6,5(R6)	596.0:	0.09	2,3,5,6
V Oph	C5,2-7,4e(N3e)	297.7	1.10	—
W Ori	C5,4(N5)	210.5:	0.13	1,3,7
GP Ori	C8,0J:(SC)ea	183.3	0.29	6
RX Peg	C4,4J(N3)	637.9	0.14	2,3,5,6
RZ Peg	C9,1e(Ne)(Tc)	437.2	1.83	3
Y Per	C4,3e(R4e)	252.9	0.42	1,3,8
TX Psc	C7,2(N0)(Tc)	852.0:	0.08	3,4
FK Pup	C6,3e(N)	252::	0.39	3,6
S Sct	C6,4(N3)	268.4:	0.07	1,3,4,5,8
Y Tau	C6.5,4e(N3)	244.1:	0.12	1,2,3,8
VY UMa	C6,3(N0)	121.8	0.03	3,4
RU Vir	C8,1e(R3ep)	436.5	1.27	3
SS Vir	C6,3e(Ne)	359.7	0.77	1,7

**The amplitudes are average values over the dataset. Also see notes to individual stars in section 3.5. (1) also analyzed by KSCM; (2) low amplitude; (3) see note in section 3.5; (4) extremely low amplitude (less than 0.10 mag.); (5) there is a gap in the data; (6) the data are sparse; (7) monoperiodic according to KSCM; (8) biperiodic according to KSCM (these periods refer to pulsation periods, not LSPs).*

3.5. Notes on individual stars

This section contains notes on individual stars in Tables 1, 2, and 3, in a single list in alphabetical order of constellation name, and in order of variable star name within each constellation.

RU And The results are somewhat uncertain because of the higher-than-average noise level.

ST And KSCM obtained periods of 181 and 338 days, but these are aliases of each other.

VX And KSCM obtained periods of 375 and 904 days; we do not find the latter period. Data after JD2441000 were used.

AQ And The period is close to a year, the amplitude is low, the data are noisy, but the result agrees with that of KSCM. We note, however, that the periods that they obtain—169 and 346 days—are close to being aliases of each other.

BI And The data were sparse and noisy.

V Aqr There is a possible 123-day period. The 689-day period reported by KSCM is an alias of the 241-day period.

S Aql There is a 245-day period whose amplitude is almost as high as that of the 147-day period. The latter is highest in the V data.

V Aql KSCM obtained periods of 400 and 215 days. We also find the latter, but there are also other peaks of comparable amplitude. There is a long time scale of about 6000 days, and the period in the table is close to an alias of this. Data after JD 2425000 were used.

GY Aql There is a 204.6-day alias, and a 230.7-day harmonic. This star does not appear to be biperiodic.

S Aur There are periods of about 368 and 590 days, with comparable amplitudes; these periods are present in the V observations also; the latter period agrees with that (590 days) of KSCM; the shorter one may be a secondary period, though the ratio 368/590 is not close to the expected ratio of about 0.5. The light curve is dominated by large, slow changes in brightness.

V Boo The amplitude is slowly decreasing. There is a possible 137-day period, as reported by KSCM.

RV Boo The 144-day period reported by KSCM is one of several low-amplitude (≤ 0.06) peaks.

S Cam Our period agrees with that of KSCM.

U Cam KSCM obtained periods of 220 and 400 days. The latter is weakly present in our results, but the strongest peak is a long secondary period of 3001 days.

RR Cam Possible 124 and 220-day periods, both reported by KSCM, with small amplitudes.

RS Cam The 160-day period reported by KSCM is one of several comparable peaks in the power spectrum. There is also a long secondary period.

ST Cam Observations after JD 2442897 were used. Results agree with those of KSCM but data are sparse, amplitude is low, and period is close to one year; it is probably spurious.

X Cnc KSCM obtained periods of 193 and 350 days, but these are close to being aliases; the amplitudes are low; and the strongest period is dangerously close to a year. V data, however, support a period of about a year.

RT Cnc Possible 700-day period, but with small amplitude.

Y CVn KSCM obtained periods of 160 and 273 days; we do not find these in our results. The amplitudes of all the peaks are low; the period in the table may be spurious.

RT Cap Because of a gap in the data, only observations after JD2434000 were analyzed; our results agree with those of KSCM.

Table 3. Periods and amplitudes of monoperiodic Oxygen Red Giants.

Star	*SpT*	*P(d)*	*A*	*Notes**
RV And	M4e	171.2	0.22	1,7
V Aqr	M6e	241.2	0.22	1,3,8
S Aql	M3-5.5e	146:	0.75	1,3,5
GY Aql	M6-8IIIe	463.4	2.05	1,3,6,7
T Ari	M6-8e	320.5	0.80	1,5,7
RS Aur	M4-6e	172.3	0.29	1,7
U Boo	M4e	201.7	0.62	1,5,7
V Boo	M6e	257.7	0.56	1,3,8
RV Boo	M5-7e	—	—	3
RR Cam	M6	—	—	3
RS Cam	M4III	90.5	0.20	1,3,6,8
RY Cam	M3III	135.2	0.19	1,7
RT Cnc	M5III	370.0:	0.11	2,3
V CVn	M4-6IIIea:	192.3	0.29	1,5,7
AA Cas	M6III	80.0	0.04	3,4
SS Cep	M5III	101.1:	0.06	1,3,4,8
DM Cep	M4	363.0:	0.05	1,4,7
RS CrB	M7	330.9	0.24	1,6,7
W Cyg	M4-6IIIe(Tc:)	132.2	0.14	1,2,3,8
TZ Cyg	M6	567.4	0.06	1,3,4,8
AB Cyg	M4IIIe	—	—	3
AF Cyg	M5-7e	94.0	0.13	1,2,3,8
AI Cyg	M6-7	—	—	1,3,7
U Del	M4-6II-III	1161	0.21	1,2,3,8
CT Del	M7	358.7:	0.12	2
CZ Del	M5	0.05	—	4
EU Del	M6.4III	62.5	0.06	1,4,7
S Dra	M7	180.1	0.10	1,2,3,5,8
RS Dra	M5e	273.6	0.40	5,6
TX Dra	M4-5e	712.0	0.14	1,8
Y Gem	M6-7e	0.10	—	2,3
SW Gem	M5III	238.4	0.08	1,3,4,7
X Her	M6e	176.9	0.06	1,3,4,8
ST Her	M6-7IIIaS	256.6	0.11	1,2,3,8
UW Her	M5e	106.8	0.10	1,2,3,8
g Her	M6III	880.0	0.06	1,3,4,7
RY Leo	M2e	159.9	0.34	1,3,7
U LMi	M6e	273.3	0.40	1,3,8
RX Lep	M6.2III	572.2	0.06	3,4,7
SV Lyn	M5III	368.2	0.07	3
SZ Lyr	M6	143.7	0.15	2,3
X Mon	M1-6IIIep	155.7	0.52	1,7
SW Mon	M4-6III	193.6	0.14	2,3
UZ Per	M5II-III	893.4	0.27	1,7
V UMa	M5-6III	197.8	0.12	1,2,3,7
Y UMa	M7II-III:	325.0	0.11	1,2,3,8
RY UMa	M2-3IIIe	287.9	0.14	1,2,3,7
RZ UMa	M5-6	260 ± 5	0.11	2
ST UMa	M4-5III	625.1	0.07	1,3,4,7
R UMi	M7IIIe	324.5	0.43	1,3,8
V UMi	M5IIIab:	72.9	0.11	1,2,3,8
SW Vir	M7III	155.4	0.20	1,3,7

**The amplitudes are average values over the dataset. Also see notes to individual stars in section 3.5. (1) also analyzed by KSCM; (2) low amplitude; (3) see note in section 3.5; (4) extremely low amplitude (less than 0.10 mag.); (5) there is a gap in the data; (6) the data are sparse; (7) monoperiodic according to KSCM; (8) biperiodic according to KSCM (these periods refer to pulsation periods, not LSPs).*

SV Cas Only data after JD2435000 were analyzed. KSCM obtained periods of 262 and 460 days. A 227-day harmonic is present. Our error in P1 is slightly large.

WZ Cas KSCM obtained periods of 187 and 373 days, but these are close to being aliases of each other. Although the period in the table is close to a year, the amplitude suggests that it is real.

AA Cas Amplitudes are very small, but the peak stands out.

S Cep There is a 243-day harmonic.

SS Cep We do not find the 340-day period reported by KSCM. The strongest peak is a long secondary period.

V Cyg There is a 209.1-day harmonic.

W Cyg There is also a 250-day period, comparable with KSCM's 240-day period, with $\Delta v = 0.07$. This period is less secure in the power spectrum of the V data.

RS Cyg There is a 211.2-day harmonic.

RU Cyg Our results are consistent with those of KSCM.

RV Cyg The amplitudes are low; the period may be spurious.

RZ Cyg Our results are consistent with those of KSCM.

SV Cyg Large gap in the data, amplitude is low, and period is close to one year. V data suggest a period of about 418 days.

TT Cyg KSCM obtained periods of 188 and 390 days, but the amplitudes of these and our periods (which include ones close to KSCM's) are less than 0.03!

TZ Cyg KSCM reported periods of 79 and 138 days. We find the second, but not the first. All peaks have $\Delta v \leq 0.06$.

AB Cyg KSCM reported periods of 429 and 513 days. We find a peak near the second, but not the first.

AF Cyg KSCM reported periods of 93 and 163 days. We find the first, and a second period of 173 ± 10 days.

AI Cyg KSCM reported a period of 146 days. We find several peaks, of comparable height, including 1864, 273, and 144 days.

AW Cyg The data are sparse, and the amplitude is small.

V460 Cyg The amplitude of the 165-day period is small, but our result agrees with that of KSCM and with an analysis of the V data.

U Del KSCM found this star to be biperiodic, with periods 110 and 580 days. The latter is an alias of the long secondary period of 1161 days, as is the period of 278 days.

S Dra KSCM reported periods of 172 and 311 days. We find periods near these, but the strongest period is actually a long secondary period.

RY Dra Observations after JD2433000 used; data are noisy, and peaks are of low amplitude. Our quoted period may be an alias of a long secondary period.

UX Dra Our highest peak is at 364.07 days, with a small amplitude, but this may be spurious. KSCM list periods of 176 and 317 days. We find periods close to these, with small amplitudes.

AH Dra Our results are consistent with those of KSCM.

AY Dra The data are sparse, but the amplitudes are high. The 130-day period may actually be a harmonic; there is a weaker 149-day period which may be the overtone.

R For The amplitude is low.

Y Gem Results highly uncertain.

SW Gem KSCM reported a period of 700 days. Our strongest peak is at 685 days, but there are also alias periods of 238 and 783 days, and a probably-spurious period of 365 days.

X Her KSCM obtained periods of 178 and 102 days; we find the shorter period in our results, but with small amplitude (≤ 0.05). The highest peaks are at periods of 650–800 days.

RR Her The shorter period is uncertain; KSCM did not list it. There is a 143.5-day alias. There is a large gap in the dataset.

ST Her KSCM obtained periods of 149 and 263 days, but these are aliases of each other, and have comparable amplitudes. The longer period produces a slightly more satisfactory alias pattern.

UW Her KSCM obtained periods of 172 and 107 days; the 107-day period is one of several with comparable small amplitude.

g Her The (low-amplitude) pulsation period is about 90 days; the 880-day period is a long secondary period. KSCM obtained periods of 887 and 90 days.

U Hya KSCM obtained a period of 791 days; we find a peak at this period, but it does not stand out; its amplitude is somewhat smaller than that of the 370-day period.

V Hya This famous star shows deep minima with a period of 6300 days; there are periods of 345 and 387 days which are aliases of this. KSCM obtained a period of 531 days which, in our results, is the highest peak which is not an alias.

Y Hya The amplitude suggests that the 363-day period may be real.

RY Hya Insufficient data for analysis.

RY Leo The power spectrum is complex. KSCM obtained close periods of 145 and 160 days.

U LMi KSCM obtained periods of 272 and 144 days; we also find the latter period; it has a much smaller amplitude than the longer period.

RX Lep The power spectrum is complex.

SV Lyn The period is probably spurious.

T Lyr There is a 340-day period which is an alias of a long secondary period; 596 days is a *possible* pulsation period.

SZ Lyr The 144-day period is supported by the V data.

SW Mon 194 days is one of several comparable peaks in the power spectrum.

W Ori KSCM listed a 208-day period, and a long secondary period of 2300 days. There are alias periods of the latter: 432 and possibly 210 days.

BQ Ori our results are consistent with those of KSCM, but are uncertain.

RX Peg Possible secondary period at 388.7 days, with an amplitude of 0.14, but this peak is noisy. Also: the strongest peak is a long secondary period. The 638-day period is supported by the V data.

RZ Peg There is a 199.0-day alias, and a 218-day harmonic. This star does not appear to be biperiodic.

Y Per KSCM obtained periods of 127 and 245 days. The former period does not stand out in our results.

SY Per Only data after JD 2447000 were analyzed. KSCM listed only a 477-day period; our 231-day period is marginally significant, but the power spectrum is noisy. There is a 207.2-day alias.

TX Psc There is a slightly weaker 255-day period which is an alias of the 852-day period, and could possibly be the true one.

FK Pup The data are sparse, but periods of about 250 and 500 days appear in both the visual and V observations.

S Sct Observations after JD 2432000 were used. The data are sparse and noisy, and the amplitudes of the peaks are small (less than 0.07), so the results are highly uncertain. KSCM obtained periods of 269 and 149 days; the latter is present in our results.

W Tau KSCM found this star to be biperiodic, with periods 265 and 243 days; we find a reasonably significant period of 127.5 days. The amplitude is low.

Y Tau KSCM obtained periods of 242 and 461 days; the latter period is present in our results, but does not stand out. The V data suggest a possible period of 320–330 days. There is also a long secondary period.

V UMa there is also a possible 107-day period.

Y UMa the power spectrum is complex. KSCM obtained periods of 164, 315 and 324 days. We also find a period of 167 ± 1 days.

Z UMa Our results are consistent with those of KSCM.

RY UMa KSCM obtained close periods of 287 and 305 days.

ST UMa The results are highly unncertain, due to small amplitude, but the 625-day period stands out.

VY UMa The 122-day period is of low amplitude, but stands out in the power spectrum; it is one of three possible periods in the V data.

R UMi KSCM obtained periods of 170 and 325 days, but these are aliases of each other.

V UMi KSCM obtained periods of 73 and 126 days. We find a broad peak at the latter period, but with amplitude less than 0.03. There is also a 770-day long secondary period.

RU Vir there is a clear period at 436 days, and a possible one at 220 days, but the latter peak is noisy. There are slow changes in mean magnitude.

SW Vir Result confirmed by V data. KSCM obtained close periods of 154 and 164 days.

RU Vul There is a large change in amplitude, half-way through the dataset. Templeton *et al.* (2005) noted that the period of this star decreased from 155 to 110 days over 65 years. The period spectrum in the first half of the data is understandably different from that in the second half. KSCM found average periods of 369 and 136 days; we find average periods of 155 and 368 days. Given the change in period and the abrupt change in amplitude, neither pair is meaningful.

4. Discussion

This study is limited by the nature and timing of the observations. As noted above, there are both alias periods (because of the yearly spacing of the observations) and possible low-amplitude spurious periods because of the Ceraski effect. These are compounded by the fact that the stars undergo only semi-regular variability, which causes the power spectrum to be more complex. If the stars are large-amplitude stars with non-sinusoidal light curves, the power spectra are also complicated by harmonics. The P/2 harmonic can masquerade as an overtone, since P1/P0 is close to 0.5 in these stars. The presence of a long secondary period (and its aliases) in about a third of the stars can also be a complication. The intrinsic variations in most of the stars are small to begin with, especially in the C stars. The C stars are also more difficult to observe, because they are very red; it is difficult to find suitable comparison stars, and different observers' eyes respond differently to red light. Furthermore: because these stars vary, on long time scales, in period, amplitude, mean magnitude, and sometimes light curve shape, our results in the tables provide only an average picture over the interval of observation.

The C stars behaved marginally differently from the M stars in terms of the relation between changing amplitude and changing period (Percy and Yook 2014). They do not behave significantly differently in terms of period ratio (Figure 3). They behave slightly differently in terms of the length of cycles of amplitude variation (Table 4), but the differences are not statistically significant. Differences might be expected, given the differences in chemical composition, and therefore of opacity and structure, in the envelopes of the stars. KSCM also looked for differences between the C and M stars, but "the photometric parameters did not allow to determine such a discrimination."

Table 4. Amplitude variations in Oxygen (top group) and Carbon (bottom group) Red Giants.

Star	*P0(d)*	*N(0)*	*L/P(0)*	*P1(d)*	*N(1)*	*L/P(1)*	*JD range*
GY Aql	463.4	1	22	—	—	—	JD 2447065+
SV Cas	455.6	1	49	239.9	3	32	JD 2435000+
RU Cyg	443.0	1.6	61	233.9	3	57	JD 2416783+
RZ Cyg	537.6	3	25	275.9	4.5	32	JD 2416792+
AH Dra	190.0	2	42	105.9	3.5	43	JD 2441000+
BQ Ori	246.5	3	34	125.8	5	39	JD 2432294+
Z UMa	189.2	2	78	98.9	5	60	JD 2427525+
S Cep	486.1	3	32	208.3	6:	32:	JD 2411073+
V Cyg	420.9	4	22	195.6	8:	22:	JD 2419700+
RS Cyg	419.0	1.5	72	195.1	4:	54:	JD 2411686+
RR Her	236.7	1.5	62	124.2	—	—	JD 2435000+
W Ori	432.4	1.5	38	210.5	3.5	34	JD 2432151+
RZ Peg	437.2	2.5	31	—	—	—	JD 2423200+
SY Per	479.2	1	23	231.5	2	24	JD 2446039+

The number of periods in the tables—especially of the C stars—which are close to one year is concerning; there is a distinct excess of stars which have periods in the range 350–380 days. Specifically: there are 11–12 C stars, and 3 M stars, whose periods are between 350 and 380 days, and which we are unsure of. The same effect is noted in KSCM, Figure 10, in which there is a pronounced peak, for the C stars, at a period of one year. Some of these periods may be spurious periods, caused by the Ceraski effect, especially if the amplitude is 0.1 magnitude or less. Reduction of the above-mentioned peak would reduce the apparent difference between the period distributions of the C and M stars in KSCM's Figure 10.

We have shown that there is still some useful science contained in the decades of visual observations of pulsating red giants in the AAVSO International Database. We have also shown the many challenges to extracting that science. The new frontier of pulsating-red-giant research is massive photometric surveys such as MACHO and OGLE. Peter Wood's (2015) paper on pulsation modes, masses, and evolution of luminous red giants is an exemplary illustration of that frontier.

5. Conclusions

Using AAVSO visual observations, and the AAVSO VSTAR time-series analysis software, we have determined improved periods for 5 biperiodic C red giants, 43 monoperiodic C red giants, 12 biperiodic M red giants, and 46 monoperiodic M red giants. We have compared the period ratios in the two biperiodic groups; they are marginally but not significantly different. We have also used VSTAR wavelet analysis to study the cyclic pulsation amplitude variations in some stars in the two groups; they are marginally different. Our main conclusion is that, because of the small amplitudes in many of these stars, and because of the presence of alias, harmonic, and spurious periods, this kind of study requires great care and caution. In particular, spurious periods, caused by the Ceraski effect, can produce an artificial peak in the period distribution, at a period of about a year.

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