Maxima and O–C Diagrams for 489 Mira Stars

Thomas Karlsson
Almers väg 19, 432 51 Varberg, Sweden; tkn@seaside.se

Abstract　Maxima for 489 Mira stars have been compiled. They were computed with data from AAVSO, AFOEV, VSOLJ, and BAA-VSS and collected from published maxima. The result is presented in a MYSQL database and on web pages with O–C diagrams, periods and some statistical information for each star.

1. Introduction

The O(bserved) minus C(alculated) diagram is a well-established technique to study changes in the periods of variable stars. O–C diagrams are sensitive to small changes in a star's period that can be signs of evolutionary changes. For long period variables of the Mira type the O–C method has been used for over 100 years; Cannon and Pickering (1909), for example, published O–C values for the maxima in their large catalogue. But it is only for a few stars certain changes have been noted. Among them are the stars R Aql, R Cen, BH Cru, LX Cyg, W Dra, R Hya, Z Tau, and T UMi (Templeton *et al.* 2005). Zijlstra and Bedding (2002) identified three types of real variations of the periods of Miras: continuously changing, where the period increases or decreases with a steady pace; sudden change, where the period dramatically changes after a long period of stability; and meandering Miras, where the period changes forth and back by up to 10% within fifty years.

A disadvantage with the O–C technique is that random intrinsic changes in the period are added over time and can create random-walk patterns, with a deviation proportional to the square root of N after N cycles. Mira stars are well known for such random cycle-to-cycle variations that make O–C diagrams for these stars hard to interpret (Eddington and Plakidis 1929; Sterne 1934). The recorded time of maximum (or whatever point in the light curve that is used for the O–C) can also differ from the true time of maximum because of sparse or inaccurate data. This type of deviation causes noise but no drift in the diagram. Given these types of noise, a baseline as long as possible is preferred to be able to see real secular or periodic changes.

For this purpose times for maximum of 489 selected Miras have been computed and collected as far back as possible. This collection is in the form of a MYSQL database and presented on web pages in the form of O–C diagrams and period diagrams together with some statistical data. The database and website is

hosted at Astronet.se and can be found at http://var.astronet.se/Mirainfooc.php or can be provided upon request to the author.

The O–C technique was chosen because it is a method that is easy to implement and works well for the older material, where the time of maximum often is the only available information, and it gives a good basic understanding of the star's behavior.

AAVSO has published a similar collection, "AAVSO Maxima and Minima of Long Period Variables, 1900–2008" (Waagen *et al.* 2010), that is also included in the database. The technique used was similar to the one used in this paper, with mean curves fitted to the observations.

2. Data collection

Of the stars in *The General Catalogue of Variable Stars* (GCVS; Kholopov *et al.* 1985) that are of type M, those were selected that were found to have enough combined data for at least twenty maxima to be determined. Two stars that fulfilled this criterion were discarded, Y Per and RZ Sco; both have a more semiregular than Mira-like behavior.

The database contains maxima of two types, those computed from and fitted to individual observations (hereafter fitted maxima) and those collected from published sources (hereafter published maxima). For the fitted maxima, data from the following organizations were used: The American Association of Variable Star Observers (AAVSO); Association Française des Observateurs d'Étoiles Variables (AFOEV); Variable Star Observers League in Japan (VSOLJ); and The British Astronomical Association-Variable Star Section (BAA-VSS).

The published maxima were used to complement the fitted to extend the timeline backward and fill in gaps among the fitted.

2.1. Published maxima

For the published maxima the main data sources are:

• "AAVSO Maxima and Minima of Long Period Variables, 1900–2008" (Waagen *et al.* 2010). This collection has information of date and magnitude for maxima and minima of 394 stars, most Miras but also some semiregulars, from observations made within AAVSO. Hereafter AAVSO maxima.

• *Annals of Harvard College Observatory*, volume 55 (Cannon and Pickering 1909). This paper contains published maxima from many different sources as they were known by 1909.

• *Annals of Harvard College Observatory*, volumes 115 and 118 (Gaposchkin and Payne-Gaposchkin 1952a, 1952b). Maxima determined from photographic plates at Harvard from 1886 to 1947, with the majority

between 1901 and 1940. Variables with a photographic magnitude brighter than 10 and known by 1936 were considered. The time for the maximum is generally rounded to nearest 5 or 10 JD.

Maxima published by various other observers found in *Astronomische Nachrichten*, *Peremennye Zvezdy*, *Mitteilungen über Veränderliche Sterne*, *Geschichte und Literatur des Lichtwechsels der Veränderlichen Sterne*, and other publications are also included.

The maxima were collected from the sources in the order above. For each source all maxima for the selected stars that were not among the fitted or in a previous source were collected. Among the lesser sources the median date was used if the same maximum was found in several sources. The same is for Cannon and Pickering (1909) where the median date was used for the maxima that have dates from several observers. AAVSO maxima use mean curves to determine the maxima; in the other sources the time for maximum light is probably the most common method.

For the AAVSO maxima a mean offset per star was calculated from the maxima that are common between the fitted and AAVSO and added to the AAVSO maxima in the database to make the two sets more consistent.

For stars with double maxima, R Cen and R Nor for example, dates for both primary and secondary maxima are sometimes published. In such cases the date that best matched the fitted was selected.

2.2. Fitted maxima

For the fitted maxima a computer program was developed in VISUAL BASIC 6. The program processes the observations and determines the time and magnitude for the maxima. The output is in form of text files that were imported to a MYSQL database. Here follows a short description of the program.

The periods from GCVS were used as preliminary periods for the stars. For each star, the two points on the mean curve where the rising and falling magnitude were equal and at a distance of 0.8 period were determined (hereafter RP and FP). RP and FP are expressed in number of days before and after maximum. They define the interval around each maximum used in the further process. Several iterations of the fitting were sometimes needed to find these points.

All available data for the selected stars were collected from the AAVSO, AFOEV, VSOLJ, and BAA-VSS. The visual and V-band data were selected and combined, and duplicate observations were removed (many observations were reported to two or more of the organizations). The remaining observations were grouped in one-day bins that were used in the further process. The times for the maxima were then determined in two steps.

For both steps the same algorithm was used to identify the individual maxima. A window with a width of one-third cycle was stepwise moved along the light curve. The brightest observation in the window was found and the

time between when the curve rose above and fell below 0.3 magnitude below this point was used as a preliminary maximum point (PM). The window width was chosen so that adjacent cycles should not interfere when looking for the PM. The observations in the interval PM–RP and PM+FP were then checked for outliers. An eighth-order polynomial was fitted to the observations and all observations 1.5 magnitudes over or under the curve were removed, except for the first and last five observations that always were retained to fix the ends of the curve. The remaining observations were then checked against a set of criteria.

In the first step a maximum had to fulfill the following criteria to be further processed:

- at least a total of fifteen points found;

- at least six points exist both before and after PM;

- the interval includes points at least 1.5 magnitudes below PM, both before and after PM;

- the gap between any two points is not greater than 0.2 of the period.

For the maxima thus detected a new eighth-order polynomial was fitted and a new PM was calculated as the maximum of the fitted curve. The observations in the PM–RP to PM+FP interval were then saved in a new collection with their offset from maximum and magnitude. After all maxima were detected the mean magnitude per day was calculated for this collection and a twelfth-order polynomial was fitted to get the mean light curve for the star, representing a stacked mean from all maxima. These mean light curves are also published and can be found on the web at http://var.astronet.se/Mirainfomax.php.

In the second step the same detection algorithm as in step one was used, but with fewer restrictions. In the first step only well documented maxima are used to build the mean curve; in the second the priority was to find as many, but still well defined, maxima as possible. The criteria for detecting maxima in the second step were:

- at least a total of twelve points found;

- at least five points exist both before and after PM;

- the interval includes points at least 1.4 magnitudes below PM, both before and after PM;

- the gap between any two points is not greater than 0.4 of the period.

The mean light curve from the first step was overlaid on each individual maximum and fitted to the observations in seven iterations. This was done by letting the observations slide step-wise alternately horizontally and vertically

against the mean curve by applying an offset to the time or magnitude. The observations that thus fell outside the RP to FP interval of the mean curve were discarded and for the rest the mean square sum of the differences between the observations and the curve was calculated. In each horizontal turn the best offset was kept and used for the next vertical turn. It was empirically tested that after seven such iterations it was generally not possible to find any better fit. The time and magnitude for the maximum point on the mean curve after the last fit was then recorded as the maximum. Figure 1 shows an example of the fitting process.

A mean curve that covers 80% of the maximum was chosen for several reasons. As a polynomial was used, it is appropriate to have a curve with not too many bends, therefore the curve was truncated before the inflection points around the minima were reached but still covering as much as possible of the curve. Maxima are usually better covered with observations than minima, hence that part of the cycle often forms the best basis for the curve. And in the cases where the period varies it is easier to fit a curve that only contains a part of a single cycle than a curve that also covers parts of the prior or next cycle. The drawback is that the fitting is not possible, or can go wrong, if the observations mainly cover only the falling or rising branch for a specific maximum. This is possible with a curve that covers more than one cycle. Examples of mean light curves are shown in Figure 2.

2.3. The database

All maxima are kept in a MYSQL database with information on cycle number (E), JD, and date for observed maximum (O), JD and date for calculated maximum (C), O–C, a reference to the source for the maximum, and magnitude at maximum (for the fitted and AAVSO maxima only). E, C, and O–C were calculated by using the elements from GCVS. In some cases manual corrections were needed to map the maxima to the right cycle number for stars with the wrong period in GCVS or having large period variations.

Finally all maxima were inspected and individual maxima that differed >40 days from their neighbors were checked. If an explanation could be found, such as a misprint in the source or the fitting was obviously wrong compared to the observations, it was corrected if possible or else that maximum was removed.

The span between the first and last maximum per star is tabulated in Tables 1 and 2.

2.4. Evaluation and analysis

A test was done to investigate possible offsets between the fitted and the datasets Canon and Pickering (1909) and Gaposchkin and Payne-Gaposchkin (1952a, 1952b). In all cases where a maximum in the fitted dataset was surrounded within five cycles before and after by maxima from one of the other datasets, or vice versa, the date for maximum was interpolated from the

surrounding maxima and compared to the surrounded maximum. The result as the mean deviation per star is presented in Figure 3. The scatter seen is probably mainly caused by the rather small number of data points per star; no significant overall trend is noticed for any of the papers, but can be present for specific stars.

The outlying stars with an offset >20 days were investigated further and are listed below. The number after each star is the number of data points.

Cannon and Pickering (1909): S Del –35 days (1), R Lyn –26 (5), and S Cas +29 (9). S Cas has five fitted maxima that coincide with a local minimum in the O–C diagram. The rejected maxima from Cannon and Pickering (1909) are near the corresponding fitted and no real offset is present. The same, but a local maximum, seems to be the case for R Lyn. S Del has one odd point.

Gaposchkin and Payne-Gaposchkin (1952a, 1952b): RX Sgr –25 (3), V Oph +24 (5), RS Sco +24 (1), S Psc –23 (1), Z Oph –23 (2), S Ori –23 (1), R Del –21 (1). The offsets are due to one or a few odd points for all stars except the carbon star V Oph where an offset is plausible.

A comparison was also made to the AAVSO maxima. Elizabeth Waagen has kindly provided the following description on how these maxima (and minima) are determined:

> For the stars with mean curves, the mean curve is superimposed on the light curve of observations, fitted to the observations by eye, and the dates and magnitudes of maximum and minimum marked. The mean curve covers one cycle and is repeated once, making a curve that shows the mean maximum and minimum twice. We find this presentation of the mean curve most helpful in fitting the mean curve to the observations, especially as we are determining both maxima and minima. Although we fit the overall curve to the data as best as is reasonable, we do not try to fit a minimum at the same time as a maximum, or two maxima at the same time. We fit only one extremum at a time.

> The mean curve is created by gathering many well-observed cycles of data of a reasonably well-behaved star, superimposing them on one another, and determining the best mean fit through the stacked data. Obviously, stars that are reasonably well-behaved from cycle to cycle will yield a more representative mean curve. It usually takes many cycles to determine how well-behaved a star is, so it is usually many years before we can determine a mean curve for a star....

> For this work to have the best outcome from year to year, it is best to have the same person doing the fitting of the mean curve and the "eyeballing" of the non-mean-curve stars. That way there is more consistency in executing the process and the smallest number of judgment errors because the person gets to know the stars by working with them year after year. (Waagen 2012)

Figure 4 shows the mean difference/P per star between the fitted and AAVSO datasets for maxima that are in both collections. The offset is in most cases rather small, less than 2% of the period for 89% of the stars or in absolute terms less than five days for 82% of the stars. The stars with an offset larger than ten days are: V Aur (–14), R Vol (–13), V Del (+12), S Cep (–12), T CVn (–12), R For (–12), RR Vir (+11), W Cas (+11), T Dra (–11), S PsA (–11), U Per (+10), and T Ari (–10). An example of a star with a large offset is shown in Figure 5.

The offsets could to some degree be explained by different shapes of the mean curves used. The mean curves for a star with a large offset and no offset are shown in Figure 2. Although the offsets for most stars are rather small, they are significant to > 2 times the standard error of the mean for 73% of the stars.

There are some other stars where the published maxima don't seem to connect very well with the fitted, including: ZZ Dra, UV Her, AZ Her, AI Per, and X Psc. These stars usually have large gaps in their sequence of maxima, so it is hard to determine if the star has had an unusual change in its period or something else is wrong.

For common stars and with date before 2008, 68.1% of the maxima are common between the AAVSO and fitted collections, 29.7% are only in the AAVSO, and 2.2% are only in the fitted collection.

3. Presentation

The data for the stars are presented on web pages and dynamically linked to the database. The main page is at http://var.astronet.se/Mirainfooc.php. For each star there is an O–C diagram with the cycle number, linked to the GCVS epoch, plotted on the x-axis and O–C in days on the y-axis. The fitted maxima are plotted with red dots, the published with blue. Under each star are some facts and statistics. By clicking on the diagrams a table with the underlying data for that star are presented with information of mean period, AAVSO offset, and all maxima with their source. An example of a diagram is found in Figure 6.

For C in the O–C, elements with $P = P_{GCVS} + \Delta P$ and Epoch = $Epoch_{GCVS} + \Delta Epoch$ were used.

The data under the diagrams contain the following information:

- The period and epoch from GCVS4.

- LC, Avg, and Per are links to the mean light curve, smoothened O–C diagram, and a period diagram.

- Mean magnitude at maximum, standard deviation, and extreme values. Only the fitted and AAVSO maxima are used for the magnitude.

- Spectral type from GCVS.

- ΔP and $\Delta Epoch$ are the linear components in the diagram, in days, when

using the elements from GCVS. These are what the GCVS elements are adjusted with in the formula above.

• ΔP/C is the parabolic component in days/cycle. This is the average change of the period each cycle during the time span of the diagram.

• E/P shows the result from the Eddington and Plakidis (1929) test. The E/P method helps to distinguish real changes from random fluctuations. Two cases are shown, with $X \leq 5$ and $X \leq 15$.

ΔP and ΔP/C also shows the standard error, SE, for the coefficients (±), t-statistics (t), and the coefficient of determination (r^2). t is ΔP or ΔP/C divided by SE. r^2 tells how well a linear or parabolic curve fit the O–C values, SE and t tell the significance of the coefficients. This provided that the deviations from the fit are normally distributed, which may not be the case for most Miras where the deviations probably have a combination of random and physical causes.

The color of ΔP and ΔP/C is green if $t \geq 5$, otherwise is it red. This is to point out stars that likely have a period that differs from that in GCVS or could be candidates for having a secular change in their period.

ΔP is clickable to show the O–C diagram with the original period elements from GCVS.

ΔP/C is also clickable to show a diagram of the residuals with the parabolic component removed. If the resulting diagram only has evenly distributed points with small amplitude it could be a sign of a star with secular change in its period.

For a discussion of the method of Eddington and Plakidis (1929), see Percy and Colivas (1999). In short, ε^2 reflects the average size of the random cycle-to-cycle variation and $2a^2$ the average difference between the true time of maximum and the recorded time. If the variation in the diagram only consists of normal distributed noise then ε^2 and $2a^2$ should be positive and r^2 near 1. If the star has secular and/or periodic changes one would expect ε^2 to have a big positive value, $2a^2$ to have a big negative value and r^2 to deviate significantly from 1.

4. Conclusions

It is demonstrated that a computer algorithm as the one described in this paper can determine maxima from observations of Mira stars with almost the same quality as those manually determined and with huge time savings. 35,000 maxima were calculated in a few hours, then a few weeks were needed to check and validate the material. The AAVSO collection, however, contains 24% more maxima for common stars. The manual method and the use of mean curves of two cycles, width seems in this aspect to be more effective. A modelling of mean curves different from that given in this paper could be a future improvement for computer-generated maxima.

In most cases the collections with older published maxima integrate well together with the fitted maxima and extend the history for the earliest discovered stars several decades or in some cases centuries back in time compared to the AAVSO collection. Also, some stars that are poorly monitored by AAVSO observers benefit from maxima published in other sources to show their earlier history.

For many stars there are gaps in the sequence of maxima, both for the fitted and other collections. This has to be accounted for when analyzing the material. The Eddington and Plakidis (1929) method, for example, is sensible for gaps in the series of maxima and can give results that do not reflect the true nature of the stars in such cases.

5. Acknowledgements

References

Campbell, L. 1955, *Studies of Long Period Variables*, AAVSO, Cambridge, MA.
Cannon, A. J., and Pickering, E. C. 1909, *Ann. Harvard Coll. Obs.*, **55**, 95.
Eddington, A. S., and Plakidis, S. 1929, *Mon. Not. Roy. Astron. Soc.*, **90**, 65.
Gaposchkin, S., and Payne-Gaposchkin, C., 1952a, *Ann. Harvard Coll. Obs.*, **115**, 1–273.
Gaposchkin, S., and Payne-Gaposchkin, C., 1952b, *Ann. Harvard Coll. Obs.*, **118**, 1–217.
Kholopov, P. N., *et al.* 1985, *General Catalogue of Variable Stars*, 4th ed., Moscow.
Percy, J. R., and Colivas, T. 1999, *Publ. Astron. Soc. Pacific*, **111**, 94.
Sterne, T. E. 1934, *Popular Astron*, **42**, 558.
Templeton, M. R., Mattei, J. A., and Willson, L. A. 2005, *Astron. J.*, **130**, 776.
Waagen, E. O. 2012, private communication.
Waagen, E. O., Mattei, J. A., and Templeton, M. R. 2010, "AAVSO Maxima and Minima of Long Period Variables, 1900–2008" (http://www.aavso.org/maxmin).
Zijlstra, A. A., and Bedding, T. R. 2002, *J. Amer. Assoc. Var. Star Obs.*, **31**, 2.

Table 1. Number of years between the first and last maximum.

Years	Number of Stars
< 40	10
40–79	45
80–119	259
120–159	159
≥160	16

Table 2. Number of cycles between the first and last maximum.

Cycles	Number of Stars
< 50	12
50–99	71
100–149	194
150–199	123
200–249	54
250–299	18
≥ 300	17
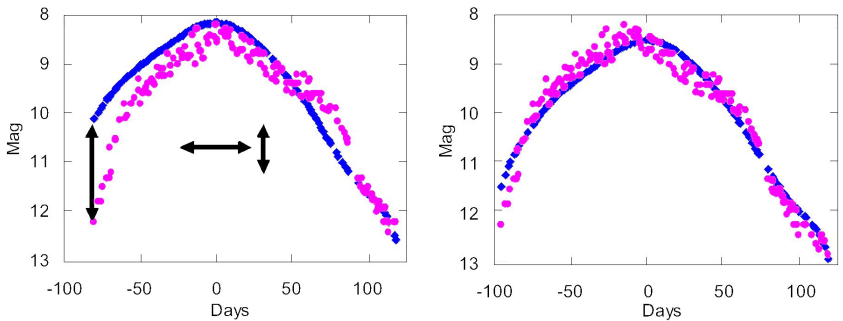
Figure 1. The fitting of the mean curve to observations of R Cam. The observations are pushed alternately left-right and up-down on the mean curve and the square sum of the deviations is calculated. To the right is the best fit. The observations are pushed 15 days to the left and the curve 0.035 magnitude down from the left image.
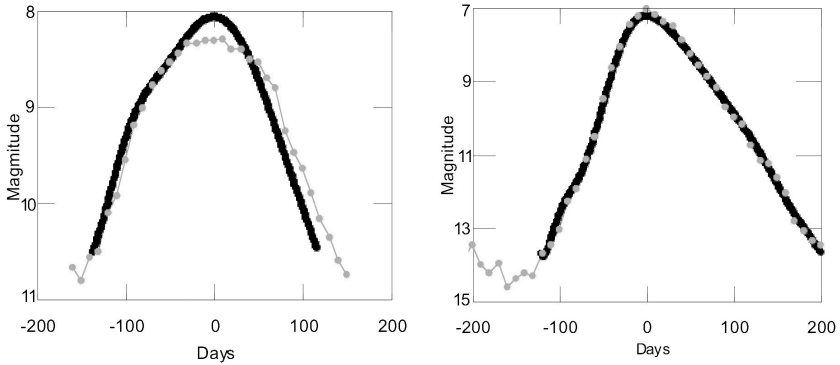
Figure 2. Two examples of mean light curves: U Per (left plot) and R And (right plot). The black lines were used for the fitted maxima, the dotted gray lines are from AAVSO data (Campbell 1955). For U Per the mean offset between the curves for the rising and falling branches is 7 days.
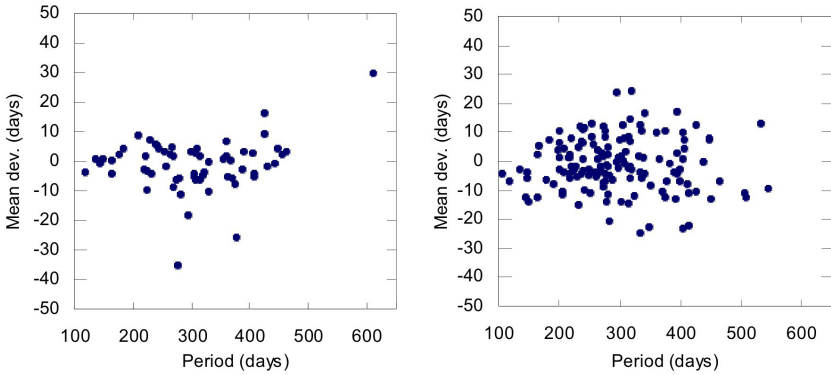

Figure 3. The mean offset for O–C values between the fitted maxima and two of the main papers. Cannon and Pickering (1909; left plot): Average offset −1.4 from 62 stars with 7.6 data points/star. Gaposchkin and Payne-Gaposchkin (1952a; right plot): Average offset −1.4 from 142 stars with 5.6 data points/star.
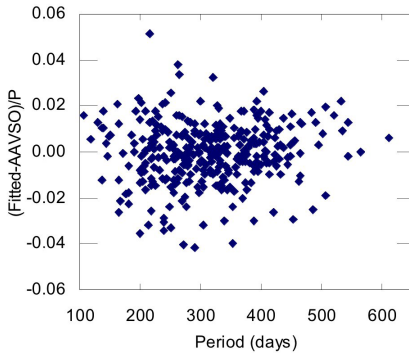
Figure 4. Average difference/P per star between the fitted and AAVSO maxima. The overall average is –0.0005 or –0.1 day from 352 stars, with on average 77.7 maxima/star.
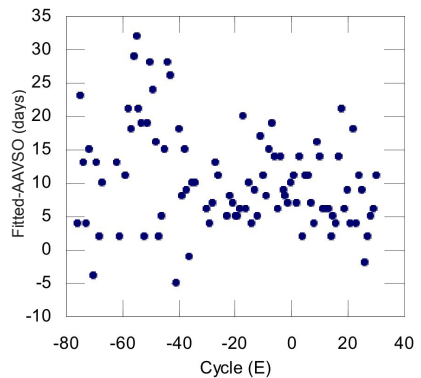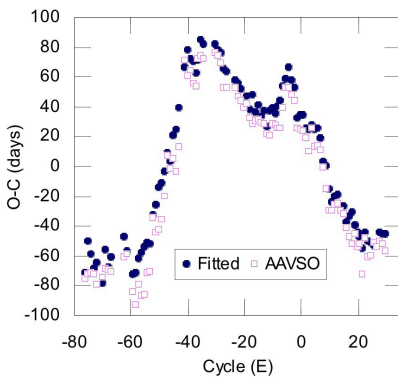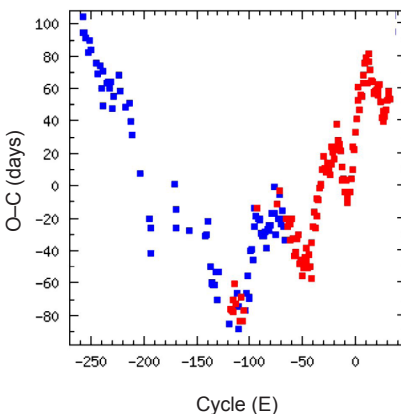

Figure 5. An example of a star (U Per) with an offset between the fitted and AAVSO maxima. The offset is 10.4 days with SEM 1.5 from 95 maxima. On the left is the O–C diagram, on the right a plot of the difference between the times of maxima. See Figure 2 for the corresponding mean curves of this star.

Period: 408.05, Epoch: 2442140   LC Avg Per
Mag at max: 5 ±0.5 (4.0-6.4)
Spectral type: S6,2e-S10,4e(MSe)
ΔP: -0.84 ±0.04 (t:19.3, r^2:0.659) ΔEpoch: -33.4
$\Delta P/C$: 0.013 ±0 (t:31.1, r^2:0.834)
E/P (X=15) ε^2: 46.2, 2a^2: -19.8, r^2: 0.995
E/P (X=5) ε^2: 35.7, 2a^2: 11.4, r^2: 0.965

Figure 6. An example of an O–C diagram (χ Cyg) with corresponding statistics. The x-axis shows cycle number and the y-axis O–C in days. Blue dots are published maxima. Red dots are fitted from observations.