Amplitude Variations in Pulsating Red Giants

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Abstract We have used long-term AAVSO visual observations and Fourier and wavelet analysis to study the long-term amplitude variations in 29 single-mode and 30 double-mode semiregular (SR) pulsating red giants, in the "long secondary periods" (LSPs) of 26 SR stars, and in 10 Mira stars. The amplitudes of the single-mode SR stars vary by factors of 2 to over 10, on time scales of 18 to 170 (median 44) pulsation periods. The amplitudes of the *individual* modes in double-mode SR stars behave similarly; the median time scale is 31 pulsation periods, with half lying between 24 and 42. The amplitudes of the two modes seem to vary independently, rather than varying in phase or anti-phase. The amplitudes of the Mira stars vary by typically factors of 1.1 to 1.3, on time scales of about 35 pulsation periods. The amplitudes of the LSPs, in most stars, vary by up to a factor of 2, on time scales of about 30 LSPs or greater. In view of the uncertainty in determining the time scales, we conclude that the time scales of the amplitude variability are similar in each of these four samples. These results should assist theorists in understanding the nature and cause of the amplitudes and their variations.

1. Introduction

The amplitudes of pulsating variable stars display a wide range of behavior with time. Most Cepheids have constant amplitudes; those with changing amplitudes, such as Polaris (Arellano Ferro 1983) are the object of special study. Many RR Lyrae stars have constant amplitudes, but some show slow, cyclic amplitude variations called the *Blazhko effect* (Kolenberg 2012) which is also of special interest. This paper deals with the amplitude behavior of pulsating red giants, especially non-Mira ones.

Pulsating red giants are classified in the *General Catalogue of Variable Stars* (GCVS; Kholopov *et al.* 1985) as Mira (M) variables if their visual amplitude is greater than 2.5 magnitudes, and semiregular (SR) or irregular (L) variables if their visual amplitude is less than that value. SR variables are subdivided into SRa and SRb (both giants), where SRb have less obvious periodicity than SRa, and SRc (supergiants). Irregular (L) variables nominally show little or no periodicity; see Percy and Terziev (2011) and references therein for comprehensive studies of a large sample of L variables. The M-SRa/SRb-L classification is an arbitrary one; there is a continuum of behavior from small-amplitude to large, and from periodic to irregular.

Many Mira variables are known to have variable amplitudes (for example, Percy *et al.* 1990); Mira itself ranges, in maximum visual magnitude, from 3 to 6. SR variables have been studied especially by Kiss *et al.* (1999) and by Percy and Tan (2013) and Percy and Kojar (2013). They found that many of these stars show at least two radial pulsation modes, and sometimes a "long secondary period" whose nature and cause are uncertain (Nicholls *et al.* 2009). These studies carried out Fourier analysis of the entire datasets, so did not study possible amplitude variations, except as noted below.

The present paper was initially motivated by the possibility that some of the double-mode SR variables would exhibit *mode switching*, in which the amplitudes of the two modes vary in anti-phase. This is shown in the interesting behavior of R Dor (Bedding *et al.* 1998), though the actual behavior is a bit more complicated than simple mode switching, as described below. In mode switching, pulsation energy may be transferred from one mode to the other, and back again, with the total pulsation energy remaining roughly constant.

Kiss *et al.* (1999) noted that the SR variable RY UMa (period 310 days) varied in amplitude on a time scale of 4000 days, and V Boo, RU Cyg, and Y Per had decreased in amplitude, as had R Dor. Kiss *et al.* (2000) followed up their earlier paper with a study of systematic amplitude variations in a sample of eight pulsating red giants. Y Per slowly reduced its amplitude, nominally changing from a Mira star to an SR variable. RX UMa, RY Leo, and V CVn apparently vary in amplitude due to beating between closely-spaced periods. In RY UMa and possibly RS Aur, the authors model the amplitude modulation in terms of the interaction between pulsation and rotation. In W Cyg and AF Cyg, they propose that the variations are due to mode switching (as in the case of R Dor).

We realized, however, that no systematic study had been carried out of the amplitude behavior of single-mode SR variables. We also analyzed a small sample of Mira stars, using the same procedure, to compare their amplitude behavior with that of the SR variables. We then studied a sample of LSPs to study their amplitude behavior, in the hope that it would shed light on their nature. Our study has been made much easier by the availability of the user-friendly time-series package VSTAR (Benn 2013)—freely available on the AAVSO website—and by the availability of the AAVSO International Database, of course.

2. Data and analysis

We used visual observations, from the AAVSO International Database (AID), of some of the single-mode and double-mode SR variables studied by Kiss *et al.* (1999), Mattei *et al.* (1997), and Percy and Tan (2013). A few of these have LSPs but, with the exception of the ones discussed below, we did not study these LSPs. The observations extend over many decades, thanks to the work of

AAVSO and other observers. They are, however, subject to some limitations. Since there are seasonal gaps in the data, there are alias frequencies in the Fourier spectra, separated from the true frequencies by an integral number of cycles per year. There is also a physiological effect, the Ceraski effect, which produces spurious one-year periods in the Fourier spectra; the amplitudes are generally a few hundredths of a magnitude at most. These alias and spurious periods are especially problematic for pulsating red giants, since their time scales are similar to the stars' true periods. There may also be small, spurious long-term variability if the visual reference stars or their assumed magnitudes have changed, though the AAVSO endeavors to minimize any such effects. Finally: we should point out that visual observation of red stars is particularly challenging because of the different color sensitivities of observers' eyes, and the fact that the visual reference stars are generally less red than the variables.

The data, extending from JD(1) (as given in the tables) to about JD 2456300, were analyzed using the VSTAR package (www.aavso.org/vstar-overview), especially the Fourier analysis and wavelet analysis routines. For the wavelet analysis, the default values were used for the decay time c (0.001) and time division Δt (50 days). The results are sensitive to the former, but not to the latter. Templeton *et al.* (2005) used c = 0.001. For the single-mode SR variables, Mira stars, and LSPs, periods between 0.9 and 1.1 times the published periods were scanned at 0.1 to 1-day resolution, using the range of JDs given in the Tables. Within these ranges, the data were sufficiently dense and continuous for analysis. Especially for the double-mode pulsators, we inspected the power spectra and wavelet plots to identify possible alias periods.

3. Results

3.1. Single-mode variables

Table 1 lists single-mode variables from Kiss *et al.* (1999). All of the stars showed variability of amplitude by typically a factor of 2 to 10 or more. The variations were not strictly periodic, but it was possible to define the approximate number N and length L of the "cycles" of increase and decrease. The amplitude plots were not sinusoids (see Figures 1–8) so the definition of the cycle lengths was somewhat subjective, but we have tried to be consistent in analyzing our four samples of stars. Figure 1 shows an example of amplitude as a function of JD. In this and the other figures, we have also shown the raw light curve. If preferred, light curves with *mean* values of the magnitudes can be constructed, using the Light Curve Generator function on the AAVSO website. The light curves show, for instance, that changes in amplitude include changes in both maximum brightness and minimum brightness. In the table: P is the period in days; JD(1) is the Julian Date of the first observation used; ΔJD is the total length of the dataset; A is the mean amplitude in magnitudes as given by Kiss *et al.* (1999); A Range is the range of amplitudes, in magnitudes, from the wavelet analysis; N is the number of cycles of amplitude increase and decrease; L is the average length in days of the N cycles; and L/P is the average length of the N cycles expressed in pulsation periods.

The ratio L/P is rather uniform, considering the uncertainty in determining L, though the ratio decreases slightly with increasing P. It ranges from 18 to 170; the median value is 44.

3.2. Mira stars

Out of curiosity, we analyzed a small number of Mira stars in exactly the same way as the single-mode SR variables; they are listed in Table 2. They were taken from the 2013 edition of the RASC *Observer's Handbook* (Chapman 2012), and cover a range of periods. The results are given in Table 2, where the columns are the same as in Table 1. The amplitudes vary by factors of 1.1 to 1.3, with one star (R And) showing only an abrupt change at the beginning of the dataset. The cycle lengths L for increase and decrease in amplitude, in units of the pulsation period P, have a median value of 35, which is not unlike that (44) for the SR variables. Figure 7 shows the amplitude variation in one star, S Hya.

3.3. Double-mode variables

The results for double-mode variables are given in Table 3, where the values of the period P in days, the mean amplitude A in magnitudes, the range R in magnitudes, and amplitude cycle length L in days and periods are given for both mode 1 (the longer) and mode 2 (the shorter). Because of the multiple periods, alias periods, and possible Ceraski periods, these stars required careful analysis, and further notes are provided in section 3.5. The amplitudes of each of the two modes were analyzed both together (that is, by scanning from below the short period to above the longer one) and separately (that is, around each period), using VSTAR.

We began by analyzing R Dor, to reproduce the results of Bedding *et al.* (1998), which we did successfully; the results are shown in Figure 2. The amplitude of the longer period is initially largest, then that of the shorter period, then the longer, and finally the shorter. But there are epochs (for example, JD 2437000–2439000) when one amplitude is largest, even though it is near the minimum in its cycle of variability, so “mode switching” is a somewhat misleading term.

In RS Cam (Figure 3), the amplitudes of the two modes are sometimes in phase, sometimes in antiphase. V CVn has two close periods, and it was not possible to separate them using wavelet analysis; Figure 4 shows the amplitude variability of the combined periods; see section 3.5 below. In RZ Cyg (Figure 5), it is clear that the amplitude of the long period varies more slowly than that of the shorter period, as is the case with R Dor (Figure 2). In W Vul (Figure 6), the amplitudes of the two modes also vary independently.

Many of the stars mode-switch in the sense that the modes alternate in *dominance*, even though they are not necessarily varying in anti-phase.

3.4. Long secondary periods

The results are given in Table 4, where the columns are the same as in Table 1, except that the pulsation period P and the long secondary period LSP, both in days, are given separately. Long secondary periods are typically an order of magnitude longer than radial pulsation periods, so our datasets of 20,000+ days are essential for this study. We found that, in most stars in our sample, the amplitudes of the LSPs varied by up to a factor of 2, though 4 of the 26 stars showed no significant variation (0.01 magnitude or less). Figure 8 shows the amplitude variation in one star, V UMi.

Even with our long datasets, few of the stars showed a full cycle of amplitude increase and decrease and, for those which did, it was obviously not possible to know whether the variation in amplitude was truly cyclic (or periodic). We could only define a "characteristic time" L for increase or decrease. For stars which showed less than a full cycle of increase or decrease, we have defined a crude lower limit for L (Table 4). We have then expressed this in units of the LSP in the last column of Table 4. The median value of L/LSP (obviously a lower limit) is about 30. It is interesting to note that this is approximately equal to the value of L/P for the single-mode and double-mode radial pulsation periods.

Several stars show no significant variability in LSP amplitude. They tend to be stars with longer pulsation periods.

3.5. Notes on individual stars

These are listed in the order that they appear in the four tables and are marked in the tables with an asterisk.

RV And: the amplitude variability is quasi-periodic.

GY Aql: note the large amplitude.

RV Boo: note the small amplitude.

T Cnc: the amplitude variability is irregular.

T Cen: the amplitude variability is irregular.

DM Cep: the data are sparse; the amplitude is small.

V460 Cyg: note the small amplitude.

U Hya: note the small amplitude.

UZ Per: only about a quarter of a cycle of amplitude variability is observed.

W Tau: note the very large range in amplitude.

R And: the amplitude is constant, except for an abrupt change at the beginning of the dataset.

W And: there is an abrupt change in amplitude at the end of the dataset.

R Tri: there is an abrupt change in the amplitude at the beginning of the dataset.

TV And: the short period is clearly separated from the aliases of the longer period, but is rather weak. The longer period varies in amplitude, but is dominant throughout the dataset.

V Aql: the overall amplitude decreases throughout the dataset.

V Boo: the overall amplitude decreases throughout the dataset, as Kiss *et al.* (1999) found, so the star has technically ceased to be a Mira star and is now an SR. The long period is dominant.

RX Boo: the Fourier spectrum is indistinct, and the amplitudes of the modes are very small at times.

U Cam: the dataset is initially sparse. The amplitude of the shorter period actually varies more slowly than that of the longer period—a rarity in our sample.

RS Cam: the data are sparse, and are only usable for the last 10,000 days.

V CVn: this is one of the stars in which Kiss *et al.* (2000) identified beating behavior. In our study, it was not possible to separate the amplitude behavior of the two close periods, 186.45 and 192.73 days, using wavelet analysis; Figure 4 shows the amplitude behavior of the combined periods. We subdivided the data into four segments, and used Fourier analysis. The amplitude of the shorter period remained approximately constant (about 0.2) but the amplitude of the longer period decreased substantially in the last quarter of the dataset—from about 0.48 to about 0.16. The beat period between the two close periods is 5,721 days, or about 30 times the mean period.

SV Cas: the data are somewhat sparse. The longer period initially has a low amplitude, but is dominant in the second half of the dataset.

RS CrB: there is some confusion between the shorter period, and an alias of the longer one, which is dominant. The data are somewhat sparse. There is a noticeable slow variation of the mean magnitude, on a time scale of 10,000 days.

W Cyg: Kiss *et al.* (2000) considered this star to undergo mode-switching, but the situation is more complicated. The shorter period has the larger mean amplitude but, for the first half of the dataset, the amplitudes are comparable and variable, so they switch dominance (“mode-switch”) several times. The amplitude of the shorter period becomes much larger toward the end of the dataset.

RU Cyg: although the shorter period has the largest mean amplitude, it is highly variable, and the two periods switch “dominance” several times.

RZ Cyg: the amplitude of the 540-day period declined slowly from 0.9 to 0.4, with four small (0.1) cycles of amplitude variation superimposed.

T Eri: note that the amplitude of the longer period is dominant and greater than 2.5; this is technically a Mira star.

RS Gem: there is some confusion between the shorter period, and an alias of the longer one. The two periods have comparable amplitudes, and do indeed mode-switch (in the sense of alternating in having the largest amplitude) several times.

g Her: there appears to be a second radial period of about 60 days (also found by Lebzelter and Kiss (2001)).

SW Mon: the periods have comparable amplitudes, and do mode-switch.

BQ Ori: the data are somewhat sparse. The modes have comparable and variable amplitudes, and switch dominance several times.

RU Per: the 170-day period may be mixed with an alias of the 316-day period. The two periods have comparable amplitudes, and do alternate in dominance several times.

S Tri: the data are sparse. The longer period is variable in amplitude, but always dominant.

V UMa: the longer period has a variable amplitude, but is almost always dominant.

V UMi: the amplitudes of the two modes are initially very small, but the amplitude of the shorter-period one becomes significant towards the end of the dataset.

SW Vir: the amplitudes of the two close periods seem to vary in phase.

W Vul: the data are sparse, and the Fourier spectrum is indistinct.

RU Vul: the amplitude decreases abruptly around JD 2439000. This decrease is associated primarily with the shorter period.

RS Cam: the data are sparse; the variability in amplitude is large, though we only observe a small fraction of the cycle of amplitude variability.

RW Eri: the data are sparse.

V Hya: this is an extremely complex star, with a large-amplitude, eclipse-like light curve. It shows bipolar outflows which appear to come from an accretion disk in an inferred binary system.

4. Discussion

Although there have been no systematic studies of amplitude changes in SR stars, there have been a few studies of small samples, including those mentioned in the introduction. Also, Percy *et al.* (2003) used long-term *photoelectric* photometry to detect cyclic amplitude variations in five small-amplitude multiperiodic pulsating red giants which are classified as SR. When the amplitude variations were re-analyzed in the same way as in this paper (and with the same uncertainties), the mean value of the cycle length L was 34P, for the twelve different periods of these five stars—consistent with the results of the present paper.

The question now arises: what is the cause of the amplitude variations, and why are the time scales consistently a few tens of pulsation periods? First: is it plausible that the actual pulsation energy increases and decreases on time scales of tens of periods? Non-linear models for pulsating red giants (Olivier and Wood 2005) and *supergiants* (Fadeyev 2012) indicate growth times of a few tens of periods, so it is possible that the pulsation energy, and hence amplitude, could vary on this time scale.

Wood (2013) has pointed out that, if a star were to dissipate its pulsation energy, then its mean brightness might increase as the amplitude was decreasing. Observing such an effect in these stars is complicated by the presence of LSPs in some of the stars, and by other long-term changes in mean brightness. SS Vir, for instance, has large changes in mean brightness, but these do not appear to correlate with the changing amplitude of the pulsation. We have looked at a number of single-mode stars with large amplitude decreases, and we find no evidence of correlations between changing amplitude and changing mean brightness—with one possible exception: SS Vir, as revealed by the photoelectric V data on this star in the last 5000 days. It shows a significant brightening as the amplitude decreases.

In the RR Lyrae stars which show the Blazhko effect, the ratio L/P is about 100; Kolenberg (2012) has described several mechanisms which might explain the effect; she leans slightly toward a model involving a resonant interaction between two radial modes.

One potentially-relevant process in SR stars might be the presence of large convection cells ("supergranular convection"). These are predicted to cause large "spots" on the photospheres of red *supergiants* (for example, Chiavassa *et al.* 2011). They have also been suggested as a cause of the "red noise" observed in the power spectra of pulsating red supergiants (Kiss *et al.* 2006) and giants (Templeton and Karovska 2009; Templeton *et al.* 2012) and also as a possible cause of the LSPs. If large convection cells caused the amplitude to vary across the face of the star, then *rotation*—which undoubtedly occurs in the stars—would modulate the observed amplitude. The constancy of L/P (at least within an order of magnitude) suggests that whatever modulates the amplitude should scale as the period, and the rotation period would do this via the radius of the star.

A simple calculation of the ratio of the rotation period to the pulsation period (the latter determined from the pulsation constant Q) shows that, for a star of two solar masses, a rotation velocity $v \sin i = 1$ km/s, a pulsation period of 300 days, and a pulsation constant Q = 0.08, gives a ratio of 33, which is very close to the median value observed. Specifically:

$$P_{rot} / P_{puls} = 32.2 \, ((m/m_{\odot})^{0.33} / Q^{0.67} \, P_{puls}^{0.33}) \, v \sin i \tag{1}$$

According to this calculation, the ratio should be larger for the first-overtone mode; indeed, the ratio of L2/P2 to L1/P1 from Table 3 has a median value of 1.33. This calculation also predicts that, for a given pulsation mode, the ratio should be smaller for larger pulsation periods. We do not find this correlation in Table 3.

Amplitude changes could thus occur if there were large convective regions which rotated around the star, and varied in time and position so as to produce the more irregular rise and fall of the amplitude. On the other hand, the rotational hypothesis does not explain why the time scales of the amplitude variations in the double-mode SR stars are different for the two modes, so we cannot tell, at this point, whether the amplitude changes are a physical effect or a geometrical effect. Bedding (2013) has suggested a plausible alternative: the amplitude variations are due to growth and decay of stochastically-excited pulsations. Christensen-Dalsgaard, *et al.* (2001) showed that SR stars have amplitude scatter which is consistent with this mechanism; they showed a simulated time series, based on this mechanism, with amplitude variations very similar to those of some of the stars in Table 1.

On a different topic: in the linear (small-amplitude) approximation, the period of a vibrating object such as a star is independent of the amplitude. It is possible, however, that the changes in amplitude that we have observed might produce observable non-linear increases in period (Bedding *et al.* 2000; Zijlstra *et al.* 2004). Because period changes produce a *cumulative* effect, they can be observable, even if they are very small. Bedding *et al.* (2000) and Zijlstra *et al.* (2004) used wavelet analysis to show that R Aql, BH Cru, and S Ori show matching amplitude and period changes. It is not known, however, whether the observed period changes are a direct result of the amplitude changes, or whether both are the result of some change in the physical properties of the stars. Also, Mira stars show apparent period changes due to random cycle-to-cycle period fluctuations (Eddington and Plakidis 1929; Percy and Colivas 1999). We have examined the period and amplitude plots for a few of our stars, and there is a general—but not exact—correspondence between the variations of the two.

5. Conclusions

We have studied the changing amplitudes of the pulsation modes in 29 single- and 30 double-mode SR stars, and a small sample of 10 Miras. We have also studied the changing amplitudes of the LSPs in 26 SR stars. Almost all of the stars show variable amplitudes and, despite the uncertainty in determining the time scales of the amplitude variability, the time scales are 30 to 45 periods in most stars in each of the four samples. This time scale is consistent with the rotation period of the stars, but the hypothesis that the changes in amplitude are connected with the rotation of the star is only partly supported by our data. There is no evidence for *systematic* mode switching in the double-mode SR stars; the amplitudes of the two modes seem to vary independently, though they may alternate in dominance. These new results raise some intriguing questions, and should provide useful information for theoreticians in understanding the pulsations and the LSPs in pulsating red giants.

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Table 1. Amplitude variability of monoperiodic pulsating red giants.

*Star**	*P(d)*	*JD(1)*	*ΔJD*	*A*	*A Range*	*N*	*L(d)*	*L/P*
RV And*	165	2428000	28300	0.30	0.20–0.60	4	7075	43
S Aql	143	2420000	36300	0.98	0.65–1.20	10:	3630:	25
GY Aql*	464	2447000	9300	2.35	1.90–2.20	1	9300	20
T Ari	320	2428000	28300	0.91	0.70–1.35	1.5	18867	59
S Aur	596	2416000	40300	0.61	0.45–0.85	2.5	16120	27
U Boo	204	2420000	49300	0.62	0.35–0.80	6	8216	40
RV Boo*	144	2434000	22300	0.09	0.05–0.15	5	4460	31
S Cam	327	2417000	39300	0.34	0.23–1.00	5	7860	24
RY Cam	134	2435000	21300	0.16	0.10–0.40	1.5	14200	106
T Cnc*	488	2417000	39300	0.34	0.23–0.47	3	13100	27
RT Cap	400	2417000	39300	0.31	0.25–0.45	1	39300	98
T Cen*	91	2413000	43300	0.62	0.50–1.20	10	4330	48
DM Cep*	367	2435000	21300	0.12	0.05–0.10	3	7100	18
RS CrB	331	2435000	21300	0.19	0.13–0.38	0.5	42600	129
AI Cyg	146	2450000	5700	0.18	0.17–0.52	1.5	3800	26
GY Cyg	143	2440000	16300	0.13	0.08–0.43	1	16300	114
V460 Cyg*	160	2435000	21300	0.08	0.04–0.14	3	7100	44
V930 Cyg	247	2442000	14300	0.72	0.30–0.70	2	7150	29
EU Del	62	2435000	21300	0.08	0.05–0.17	4	5325	86
SW Gem	700	2427500	28800	0.10	0.05–0.35	0.5	57600	82
RR Her	250	2435000	21300	0.54	0.10–0.70	1	21300	85
RT Hya	255	2415000	41300	0.20	0.20–1.00	2.5	16520	65
U Hya*	791	2420000	36300	0.06	0.04–0.16	0.5	72600	92
X Mon	148	2415000	41300	0.59	0.25–0.85	4	10325	70
SY Per	477	2446000	10300	0.89	0.67–0.92	1	10300	22
UZ Per*	850	2448000	8300	0.25	0.23–0.29	0.25	33200	39
W Tau*	243	2415000	41300	0.27	0.10–1.50	1	41300	170
V UMa	198	2420000	36300	0.19	0.15–0.50	6.5:	5585	28
SS Vir	361	2420000	36300	0.81	0.60–1.15	3.5	10371	29

**Note in section 3.5*

Table 2. Amplitude variability of a small set of Mira stars.

*Star**	*P(d)*	*JD(1)*	*ΔJD*	*A Range*	*N*	*L(d)*	*L/P*
R And*	409	2420000	36300	constant	—	—	—
W And*	397	2420000	36300	2.95–3.15	6	6050	15
R Aur	459	2420000	36300	2.05–2.80	2	18150	40
T Cam	374	2420000	36300	2.15–2.75	3	12100	32
V Cnc	272	2420000	36300	2.00–2.45	3.5	10371	38
T Cas	445	2410000	46300	1.40–1.95	3:	15433	35
U Cet	235	2420000	36300	2.05–2.65	3.5	10371	44
S Hya	257	2420000	36300	2.10–2.55	4.5	8067	31
R Tri*	266	2420000	36300	2.40–2.65	3	12100	45
R UMa	302	2420000	36300	2.55–2.90	5	7260	24

**Note in section 3.5*

Table 3. Amplitude variability of double–mode semiregular variable stars.

*Star**	*P1/A1, P2/A2*	*JD(1)*	*L1/P1*	*R1*	*L2/P2*	*R2*
TV And*	112/0.71, 62/0.31	2427500	37	0.15–0.65	42	0.10–0.40
V Aql*	400/0.10, 215/0.11	2423000	21	0.28–0.04	26	0.31–0.04
V Boo*	258/1.20, 134/0.47	2419000	36	0.10–1.50	40	0.15–0.50
RX Boo*	305/0.14, 162/0.17	2434000	15	0.02–0.19	20	0.03–0.22
U Cam*	400/0.09, 220/0.09	2427500	29	0.04–0.19	53	0.05–0.23
RR Cam	223/0.09, 124/0.10	2427500	20	0.10–0.30	29	0.08–0.28
RS Cam*	160/0.15, 90/0.12	2445000	25	0.07–0.28	17	0.15–0.50
V CVn*	194/0.42, 186/0.13	2433000	30	0.07–0.17	42:	0.11–0.58
SV Cas*	460/0.48, 262/0.32	2435000	62	0.25–0.75	23	0.20–0.85
WZ Cas	373/0.16, 187/0.09	2430000	24	0.07–0.28	24	0.06–0.29
T Col	225/3.95, 116/0.36	2415000	61	1.75–2.40	178	0.35–1.15
RS CrB*	332/0.34, 183/0.10	2435000	16:	0.03–0.16	13:	0.03–0.13
W Cyg*	237/0.23, 131/0.54	2420000	31	0.07–0.40	56:	0.07–0.43
RU Cyg*	434/0.30, 235/0.69	2420000	42	0.05–0.42	52	0.11–0.76
RZ Cyg*	537/0.63, 271/0.86	2415000	19	0.90–0.40	33	0.40–1.40
R Dor	330/0.15, 176/0.10	2430000	100	0.13–0.48	95	0.05–0.35
S Dra	311/0.12, 172/0.12	2420000	78:	0.10–0.55	141	0.05–0.75
TX Dra	136/0.13, 77/0.31	2432000	30	0.06–0.20	33	0.05–0.37
T Eri*	254/4.49, 132/0.25	2428000	28	2.00–2.50	43	0.20–0.95
RS Gem*	271/0.25, 148/0.22	2427000	18	0.15–0.65	40	0.11–0.65
g Her*	90/0.16, 60/0.05	2432000	25	0.03–0.19	37	0.03–0.15
SW Mon*	194/0.40, 103/0.30	2428000	24	0.13–0.46	23	0.13–0.47
BQ Ori*	240/0.14, 127/0.10	2434000	23	0.12–0.40	25	0.09–0.51
RU Per*	329/0.38, 170/0.42	2428000	29	0.05–0.60	37	0.06–0.43

Table continued on next page

Table 3. Amplitude variability of double–mode semiregular variable stars, cont.

*Star**	*P1/A1, P2/A2*	*JD(1)*	*L1/P1*	*R1*	*L2/P2*	*R2*
S Tri*	250/1.06, 131/0.29	2440000	33	0.25–0.70	31	0.17–0.46
V UMa*	199/0.49, 109/0.16	2420000	15	0.07–0.38	74	0.06–0.52
V UMi*	127/0.10, 73/0.14	2432000	21	0.03–0.17	33	0.04–0.29
SW Vir*	164/0.13, 154/0.20	2434000	34	0.05–0.55	29	0.10–0.62
W Vul*	242/0.34, 126/0.35	2436000	22	0.10–0.45	30	0.10–0.37
RU Vul*	369/0.13, 136/0.11	2428000	38	0.03–0.23	104:	0.06–0.80

**Note in section 3.5*

Table 4. Amplitude variability of long secondary period in SR variables.

*Star**	*P(d)*	*LSP(d)*	*A*	*JD(1)*	*A Range*	*ΔJD*	*L(d)*	*L/LSP*
U Cam	400, 220	2800	0.13	2427000	0.15–0.18	28800	≥115200	≥41
RS Cam*	160, 90	966	0.17	2445000	0.09–0.25	11300	45200	47
ST Cam	372, 202	1580	0.10	2435000	constant	21300	—	—
X Cnc	350, 193	1870	0.08	2433000	0.03–0.05	23300	46600	25
Y CVn	273, 160	3000	0.08	2432000	constant	24300	48600	—
V465 Cas	97	898	0.16	2440000	0.13–0.25	16300	≥32600	≥36
AF Cyg	163, 93	921	0.08	2425000	0.05–0.20	31300	41733	45
AW Cyg	387	3700	0.10	2435000	constant	21300	—	—
V927 Cyg	229:	2900	0.25	2450000	constant	2700	—	—
U Del	110:	1146	0.21	2430000	0.14–0.26	26300	43833	38
RY Dra	200–300	1150	0.20	2435000	0.08–0.14	21300	28400	25
TX Dra	137, 77	706	0.10	2432000	0.09–0.19	24300	24300	34
Z Eri	74	729	0.15	2432000	0.08–0.19	24300	18692	26
RW Eri*	91	950	0.15	2435000	0.09–0.19	21300	≥42600	≥26
TU Gem	215	2406	0.10	2433000	0.10–0.12	23300	≥46600	≥19
g Her	90	887	0.20	2432000	0.14–0.22	24300	32400	37
UW Her	172, 107	1000	0.09	2435000	0.09–0.12	21300	14200	14
V Hya*	531	6400	1.22	2416000	1.09–1.16	40300	—	—
S Lep	97	856	0.24	2435000	0.19–0.35	21300	28400	33
W Ori	208	2390	0.15	2433000	0.18–0.23	23300	≥58250	≥24
V431 Ori	273:	2400	0.18	2440000	0.17–0.20	16300	≥65200	≥27
BD Peg	78:	3300	0.18	2435000	0.11–0.21	21300	≥85200	≥26
τ^4 Ser	111	1240	0.10	2435000	0.07–0.12	21300	21300	17
ST UMa	615	5300	0.11	2432000	no result	24300	—	—
V UMi	126, 73	737	0.06	2433000	0.03–0.10	23300	23300	32
SW Vir	164, 154	1700	0.15	2434000	0.10–0.16	22300	27875	16

**Note in section 3.5*

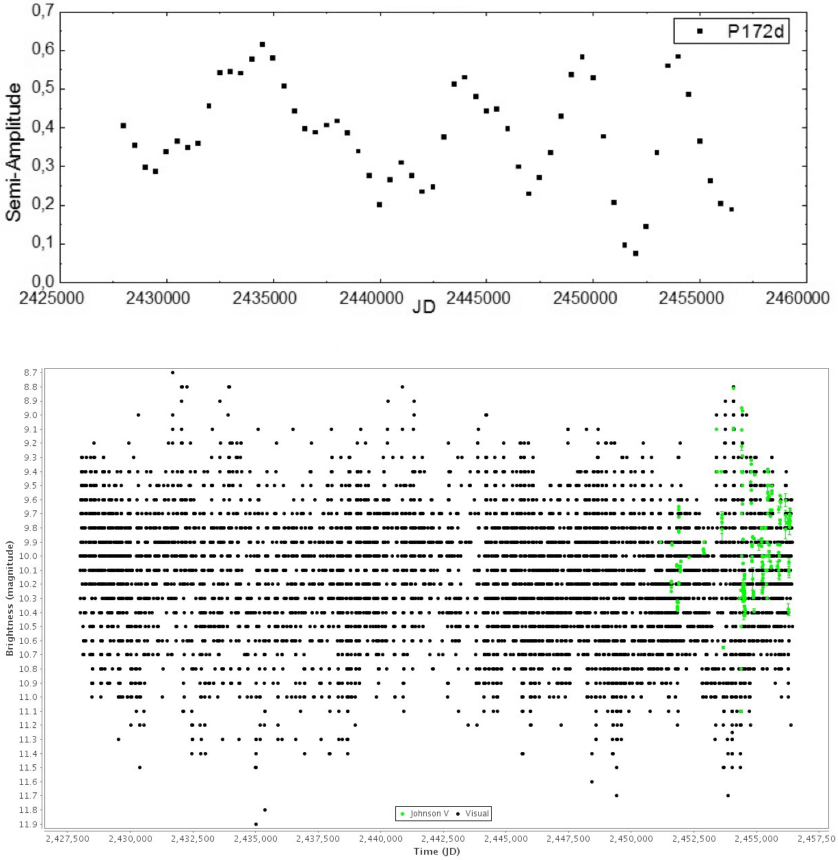


Figure 1. The changing amplitude of the single-mode semiregular variable RV And, determined by wavelet analysis (upper plot). The pulsation period is 172 days. The amplitude varies by a factor of six, and there are approximately four cycles of increase and decrease. The light curve from the AAVSO International Database (AID) is shown in the lower plot.

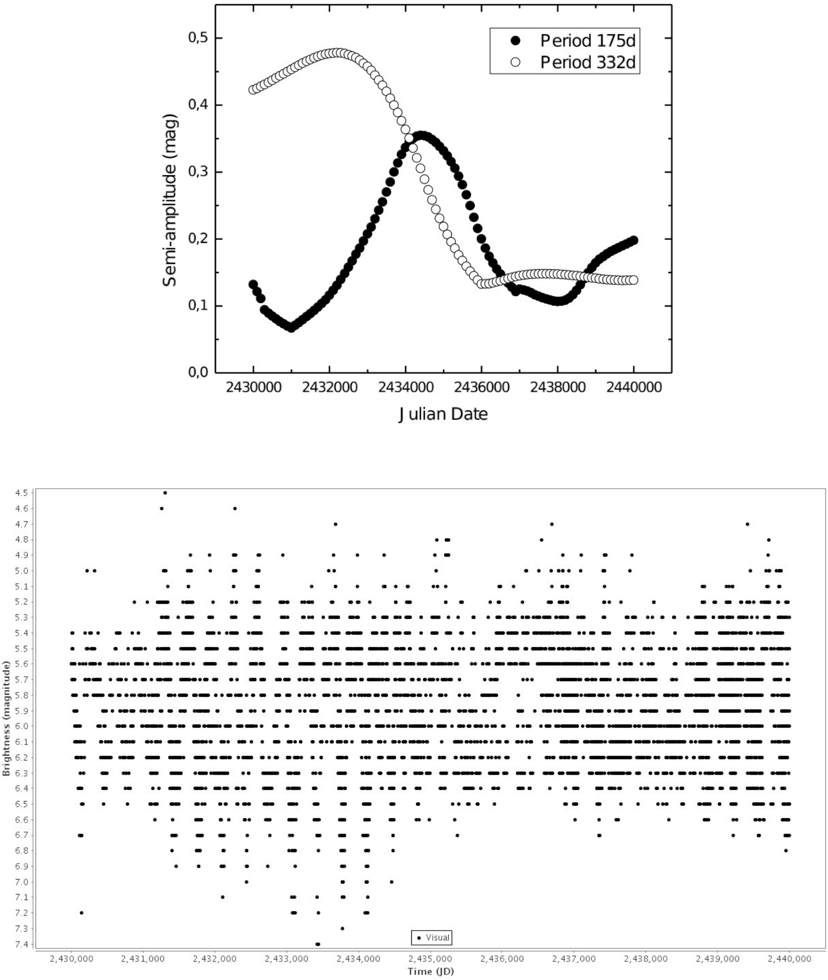
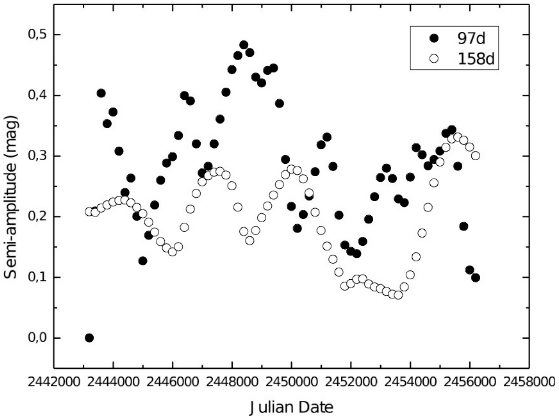


Figure 2. The changing amplitudes of the two pulsation modes in the double-mode SR star R Dor (upper plot). The amplitude of the shorter period varies more rapidly than that of the longer period. The star switches modes in the sense that the two modes alternate in dominance, but the longer period dominates, the second time, when it is at minimum amplitude. The light curve from the AID is shown in the lower plot.



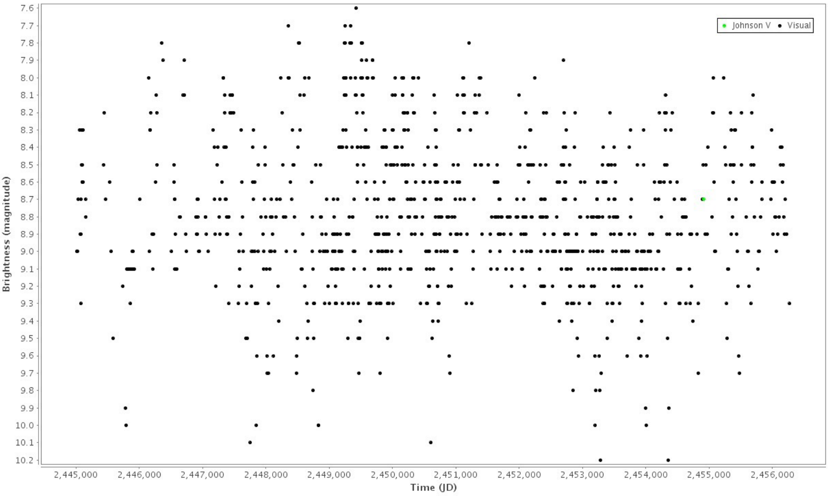


Figure 3. The changing amplitudes of the two pulsation modes in the double-mode SR star RS Cam (upper plot). The shorter period varies in amplitude more rapidly than the longer one. The light curve from the AID is shown in the lower plot.

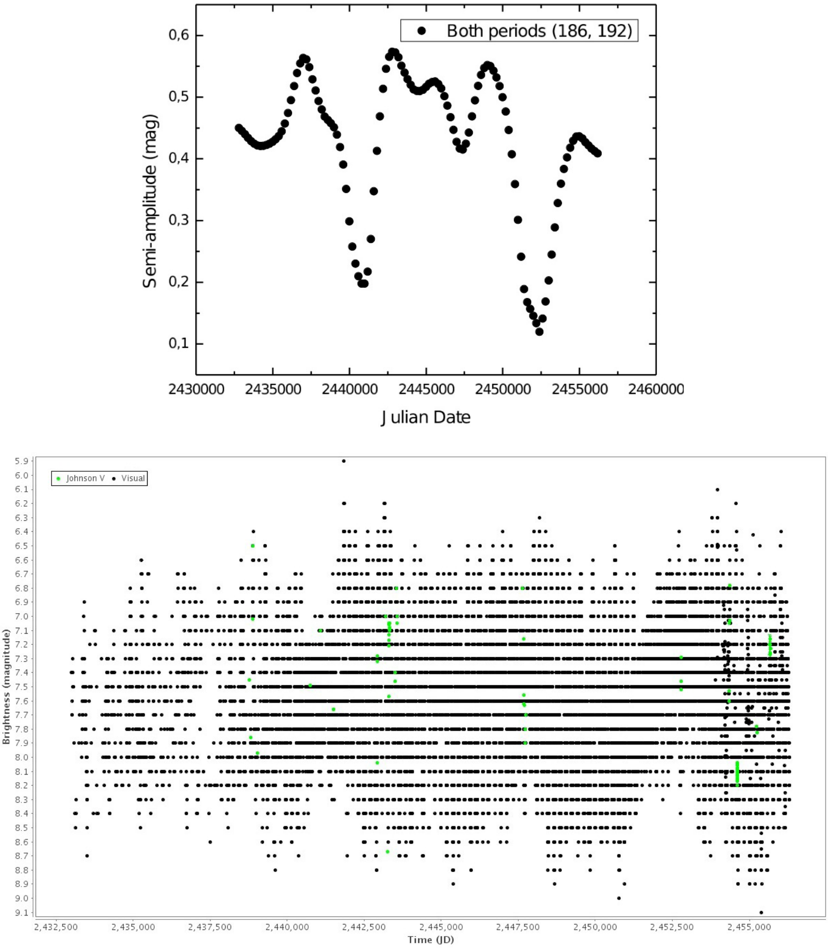
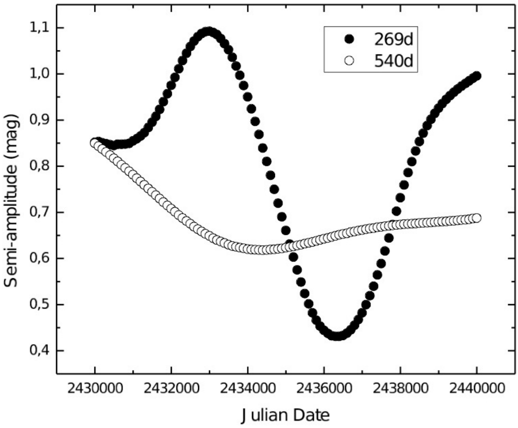


Figure 4. The changing amplitudes of the two pulsation modes in the double-mode SR star V CVn (upper plot). The amplitude variations of the two close periods could not be individually followed. The light curve from the AID is shown in the lower plot.



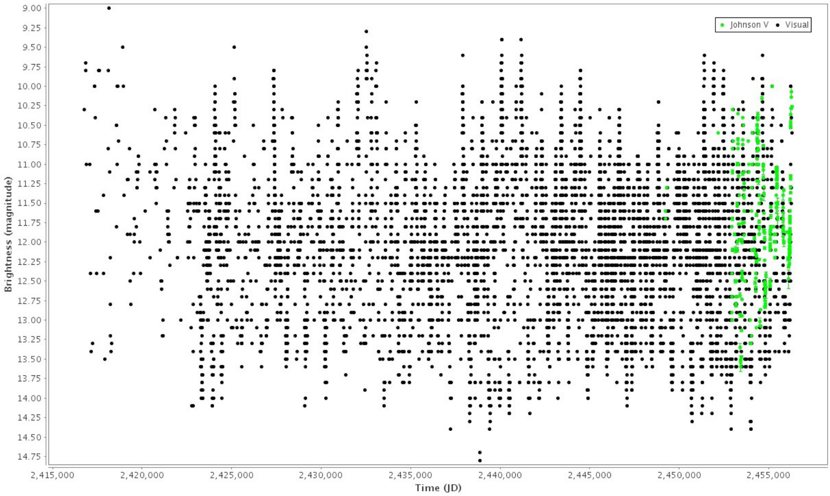


Figure 5. The changing amplitudes of the two pulsation modes in the double-mode SR star RZ Cyg (upper plot). The amplitude of the shorter period varies more rapidly than that of the longer one. The light curve from the AID is shown in the lower plot.

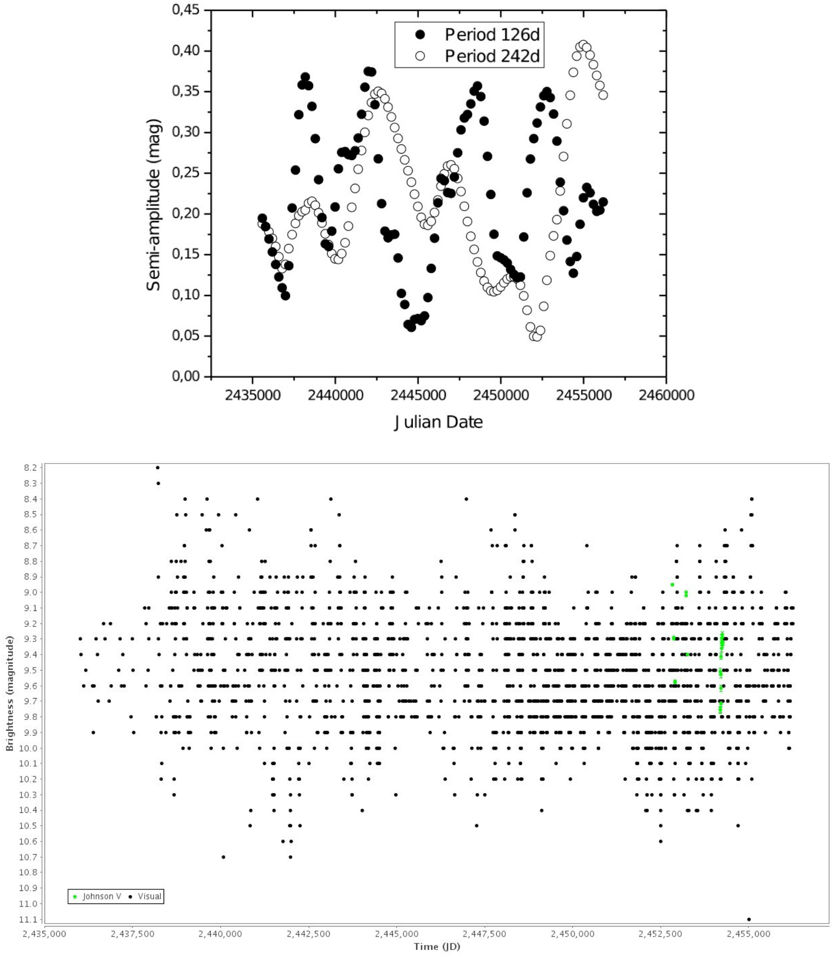


Figure 6. The changing amplitudes of the two pulsation modes in the double-mode SR star W Vul (upper plot). Note that the amplitude variations of the two modes are sometimes in phase, sometimes in anti-phase, and sometimes neither. The light curve from the AID is shown in the lower plot.

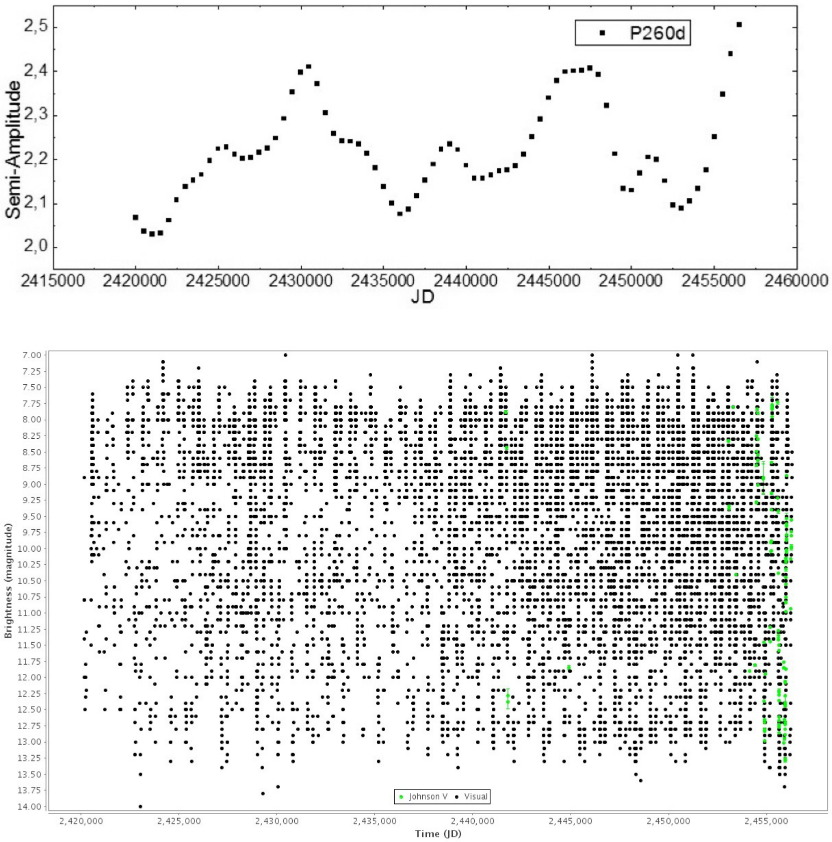


Figure 7. The changing amplitude of the Mira star S Hya, determined by wavelet analysis (upper plot). The pulsation period is 260 days. The light curve from the AID is shown in the lower plot.

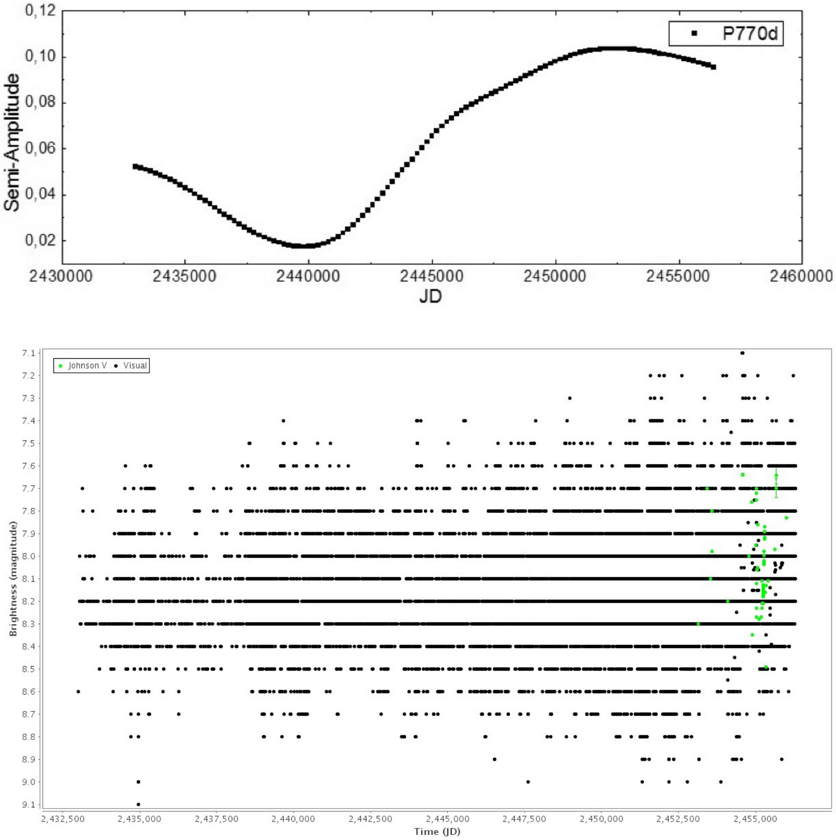


Figure 8. The changing amplitude of the long secondary period of the semiregular variable V UMi, determined by wavelet analysis (upper plot). The long secondary period is 770 days. It is not possible to know whether the amplitude variation is actually cyclic; only a fraction of a "cycle" can be observed. The light curve from the AID is shown in the lower plot.