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NOVAE THROUGH THE (CONVEX) LOOKING GLASS

by

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Abstract

Properties of the inverse hyperbolic sine are described and are exploited to construct a new form of nova light curve and to display the period-amplitude relation for recurrent novae.

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Figure 1 shows at a glance the photometric history of V1500 Cyg (Nova Cygni 1975) from 1952, when it was invisible on the Palomar Sky Survey charts, to the end of November, 1975. In Figure 2 the light curve of V1500 Cyg is compared with the light curves of four fast novae from the time of outburst to the time when they settled back to constant (or nearly constant) light many years later. In spite of the enormous interval covered by these diagrams, no detail of the light curve was lost around the time of maximum, when the development of the novae was at its fastest. This was made possible by using in the abscissa not time, but a very useful mathematical function of the time interval $\Delta t = t - t_{max}$ elapsed from the instant of maximum light. This function is $\sinh^{-1} (\Delta t/2)$, i.e., the "inverse hyperbolic sine" of $\Delta t/2$. As can be seen from Table 1, it is zero when $\Delta t = 0$; when Δt is small, it is nearly proportional to Δt itself, but as Δt grows larger, it approaches the natural logarithm of Δt. It also is symmetrical about zero. This can be easily understood if we consider the defining expression

$$\sinh^{-1} x = \ln (x + \sqrt{x^2 + 1}) \; .$$

Table 1

The Inverse Hyperbolic Sine

x	$\ln(x + \sqrt{x^2+1})$	$\ln(2x)$	x	$\ln(x + \sqrt{x^2+1})$	$\ln(2x)$
-4.0	-2.095	..	1.0	0.881	0.693
-2.0	-1.444	..	2.0	1.444	1.386
-1.0	-0.881	..	4.0	2.095	2.079
-0.5	-0.481	..	8.0	2.776	2.773
-0.25	-0.247	..	16.0	3.467	3.466
0.00	0.0	$-\infty$	32.0	4.159	4.159
0.25	0.247	-0.693	64.0	4.852	4.852
0.50	0.481	0.0	128.0	5.545	5.545

Although very useful in a variety of applications, hyperbolic functions have never enjoyed the popularity of logarithms. Today, however, they are featured on pocket calculators, so there is no excuse for not making wider use of them.

In Figure 1, I have plotted all the relevant, photographic observations of the nova before its big outburst on August 28/29, as far as they have come to my knowledge. They are summarized in Table 2.

Table 2

Pre-eruption Magnitudes of V1500 Cyg

Date (U.T.)	Mag.	Photograph	Observer
1952 July 20	(21 B	Palomar Sky Survey	de Vegt *et al.* (1975) Samus (1975)
1952 July 20	(20 R	"	"
1972 -	(19 B?	Baldone	Alksne (1975)
1974 Dec.	(17.9 B?	"	"
1975 Aug. 5/6	16.0 V	"	"
1975 Aug. 7/8	17.6 B	"	"
1975 Aug. 12/13	17.0 B	Crimea Obs.	Samus (1975)
1975 Aug. 24/25	13.5 R	Baldone	Alksne (1975)
1975 Aug. 28.732	13.5 B	Dushanbe	Suyarkova (1975)

The last five observations can be reconciled by assuming that during the month of August, 1975, the nova had a color index (B - V) of about $+1^{m}.0$ and was steadily increasing in brightness, as indicated by the slanting straight line leading to the big outburst in Figure 1. It must be remembered that a linear increase in our diagram corresponds to an accelerated increase in a diagram with ordinary time in abscissa. Particularly important is the last observation in the table, which caught the nova just as it was about to start on its big outburst. Only 7.7 hours later the nova appeared of $8^{m}.4$ on a panchromatic film exposed by Garnavich (1975) in Maryland. The photographs of this observer and those of Mayer (1975) in California, taken for a meteor program, gave a coverage of the rise of the nova almost to its maximum better than that of any previous nova. In the early phase of the outburst, between Aug. 28.8 and Aug. 29.0, the brightness was increasing at the rate of about 17^{m} per day; around Aug. 29.25 the rate had decreased to 13^{m} per day.

A slow increase in brightness before the big outburst is not unprecedented in novae: before its steep maximum of $3^{m}.0$ on Jan. 29, 1963, V533 Herculis had been rising throughout the year 1962 from $13^{m}.7$ to $11^{m}.8$ (Robinson 1975). What is extraordinary in V1500 Cyg is the spectacular increase of more than 19 magnitudes from its quiescent stage. Most novae have amplitudes between 11^{m} and 13^{m}. Except for CP Puppis (1942), the largest observed amplitude is that of V476 Cygni (1920), $14^{m}.0$. It is understandable, therefore, that some observers should have suggested that the object is a galactic supernova of type II. I have pointed out that the spectrum of the object at maximum light was more typical of a nova than of a supernova and that its behavior recalled that of CP Puppis, which rose at least $16\overset{\mathrm{m}}{.}5$ to maximum, and I have suggested that both objects might be virgin novae, erupting for the first time (Jacchia 1975). The main reason for this suggestion is that CP Puppis, which rose from fainter than 17^{m} (no prenova image could be found on any photograph) to $0\overset{\mathrm{m}}{.}6$, after an exceptionally fast decline that lasted a few days and then a more regular decay, seems to have finally settled during the past several years at $14\overset{\mathrm{m}}{.}5$, i.e., at a level several magnitudes brighter than its prenova stage. As is well known, prenova and postnova magnitudes are essentially the same, within a few tenths of a magnitude, for all other novae for which reliable data exist (Robinson 1975), indicating that the amount of matter ejected during an eruption is but a small fraction of the mass of the star.

Kukarkin and Parenago (1934) showed that in cyclic eruptive variable stars (U Geminorum and Z Camelopardalis stars and recurrent novae), the amplitude *A* of the variation, expressed in magnitudes, increases linearly with the logarithm of the cycle length *P*. I have tried to improve the relation on the basis of the most recent data, using only well observed stars: 38 U Gem stars, 10 Z Cam stars, and 5 recurrent novae. In the very few cases where only photographic data were available, the photographic (blue) amplitude was reduced to visual by multiplying it by 0.9. Average typical amplitudes were used for each star instead of observed extremes, as had been done by other investigators (Payne-Gaposchkin 1964; Glasby 1970), to avoid incorporating exceptional extremes or observational errors into the data. The result is shown in Figure 3, where again I have used $\sinh^{-1}(P/2)$ instead of $\log P$ in the abscissa. The reason for this choice should become obvious, seeing how neatly the straight line that fits the data points passes through the origin. When we use a logarithmic relation between A and $\log P$, we find that, if we extrapolate it to very small values of P, the computed amplitude becomes negative, reaching $-\infty$ for $P = 0$. This is clearly a physical impossibility. Actually, in the observed range of P, from 11 to 50,000 days, there is hardly any difference between $\ell n\, P$ and $\sinh^{-1}(P/2)$, so the choice of the function P to be taken as abscissa was more a matter of principle than of necessity.

From Figure 3, we derive

$$A = 0.81 \sinh^{-1}(P/2) = 0.81\, \ell n\, P$$

or (1)

$$P = 2 \sinh(1.23\, A) \simeq e^{1.23A} ,$$

when P is expressed in days. Since this relation is valid for recurrent novae, it stands to reason that it should also be valid for ordinary novae. In that case, the average interval between outbursts would range from 7500 years for a nova with an amplitude of 12^{m} to 90,000 years for one with $A = 14^{\mathrm{m}}$. These are relatively short time intervals compared with characteristic times involved in stellar evolution, so it appears highly probable that most of the novae that have been observed have had previous outbursts. But there must be a first outburst for every nova, and it does not seem impossible that we may have witnessed a couple of them among the 160 galactic novae that have been observed so far.

Let us now consider Figure 2. As can be seen, in this type of diagram the approach to the postnova, constant-light stage appears to be rather abrupt and it is easier to determine the time t_c at which constant light is reached. I find that for most novae the descent time $\tau = t_c - t_{max}$, which for all practical purposes is identical with the total duration D of the nova stage, can be reasonably well represented by

$$\tau = 2 \sinh (\tfrac{2}{3}A) \simeq e^{2A/3} \ . \qquad (2)$$

According to this formula, τ = 31 years when A = 14^m and 8 years when A = 12^m. For the recurrent nova T CrB, with A = $8\overset{m}{.}8$, the computed value of τ is 1.0 year, in excellent agreement with observations.

Combining equations (1) and (2), we obtain

$$\tau/P \simeq P^{-0.46} \ . \qquad (3)$$

This formula predicts that in U Gem and Z Cam the ratio of the duration of the maximum to the length of the cycle should decrease with increasing P: for P = 11 days we should have τ/P = 0.33; for P = 30 days, τ/P = 0.21; and for P = 100 days, τ/P = 0.12. These predictions are in excellent agreement with the observational data.

For most ordinary fast novae the descent from maximum takes some 10 to 30 years. According to equation (2) it should have taken CP Puppis more (and possibly much more) than 150 years to return to the prenova magnitude, fainter than $17\overset{m}{.}0$. Instead it stopped fading after 22 years, when it reached $14\overset{m}{.}5$. If this is going to be the normal minimum of the nova between outbursts from now on, we have A = $13\overset{m}{.}9$, for which equation (2) gives τ = 29 years. Similarly it should take V1500 Cyg more than 3300 years to become fainter than 21^m. It is a good bet that it will stop fading long before that, at a much higher level of brightness than its prenova stage.

A few more words about the data plotted in Figure 1. The photoelectric observations show a much larger scatter (up to $0\overset{m}{.}25$) than would be expected from their internal accuracy (of the order of $0\overset{m}{.}01$). This can be ascribed to the cumulative effect of two causes: inconsistencies in the assumed magnitude of the comparison stars and rapid variations in the light of the nova. Concerning comparison stars, it must be noted that several photoelectric observers used the nearby star f^2 Cygni, which is reddish (spectrum K5) and has been on the list of suspected variables since 1883. As to rapid variations, several observers found that on September 9/10 the brightness of the nova was fluctuating with an amplitude of $0\overset{m}{.}1$ and a period of about 3.3 hours, which should be doubled if the variations were caused by an orbiting companion.

REFERENCES

Alksne, Z. K. 1975, Astronomicheskii Tsirkulyar, 889.

de Vegt, C., Gehlich, U. K., and Kohoutek, L. 1975, I.A.U. Circular, 2826.

Garnavich, P. 1975, I.A.U. Circular, 2848.

Glasby, J. S. 1970, The Dwarf Novae (New York: Elsevier), pp. 28-

Jacchia, L. G. 1975, I.A.U. Circular, 2828.

Kukarkin, B. V., and Parenago, P. P. 1934, *Veränderliche Sterne*, *4*, 251-254.
Mayer, B. 1975, *I.A.U. Circular*, 2848.
Payne-Gaposchkin, C. 1964, *The Galactic Novae* (New York: Dover), pp. 250-251.
Robinson, E. L. 1975, *A. J.*, *80*, 515.
Samus, N. N. 1975, *Astronomicheskii Tsirkulyar*, 889.
Suyarkova, O. 1975, *Astronomicheskii Tsirkulyar*, 889.

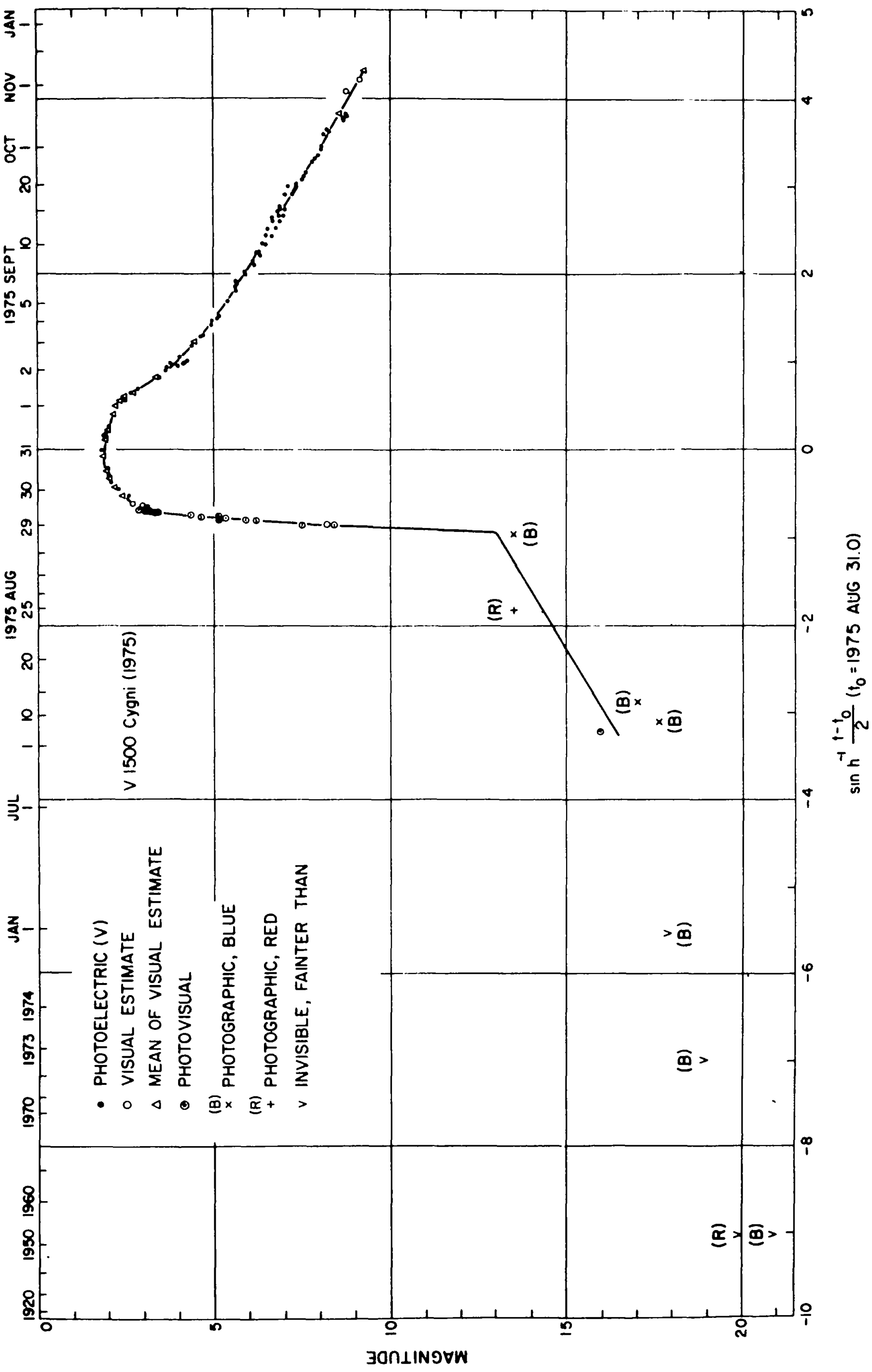

Figure 1. Light curve of V1500 Cygni in a modified time scale described in the text.

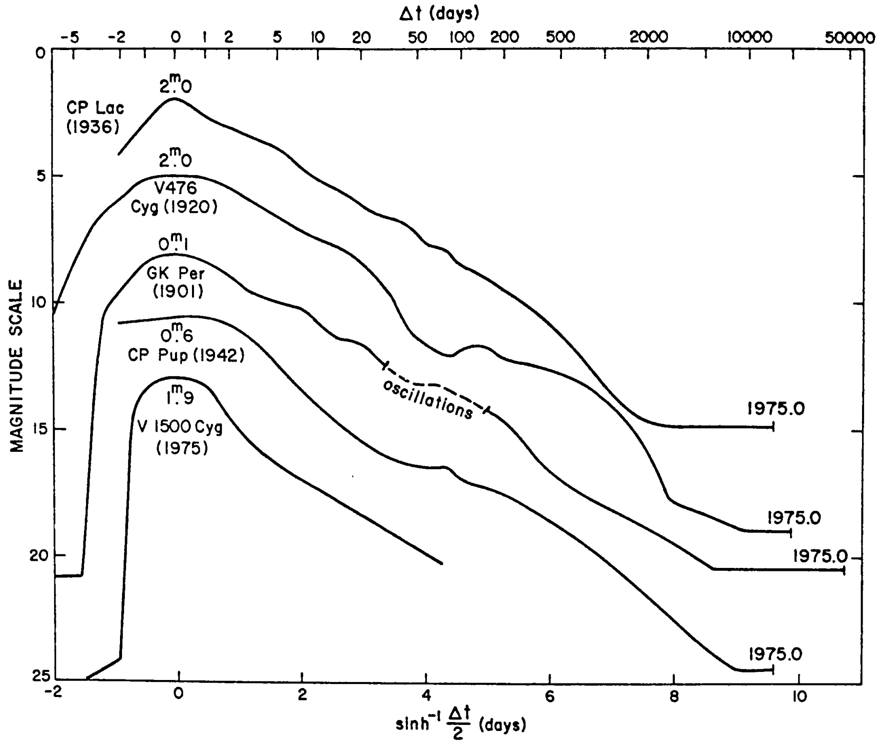

Figure 2. The light curve of V1500 Cygni compared with the light curves of four fast novae, in the same modified scale as in Figure 1.

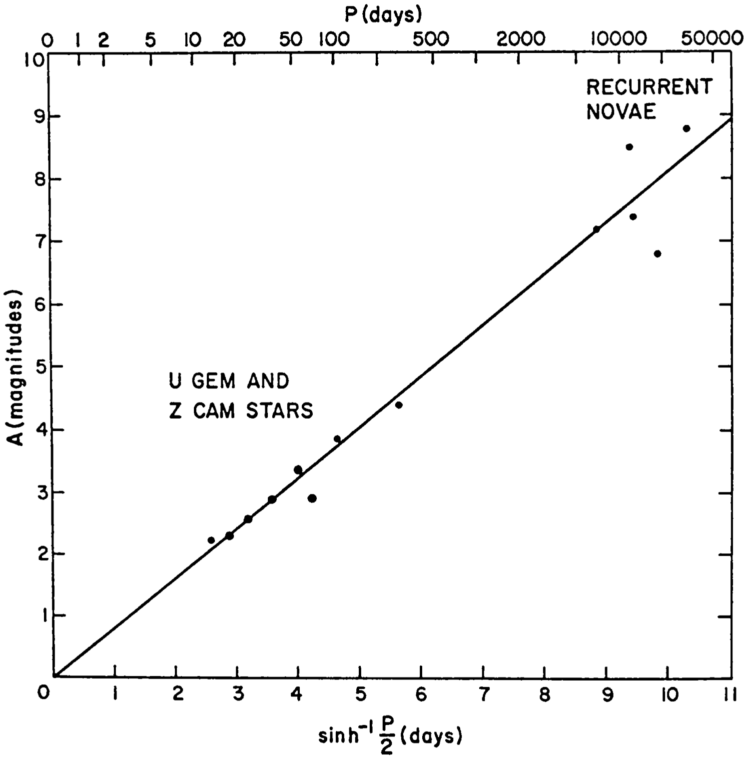

Figure 3. The relation between visual amplitude and length of cycle for eruptive variable stars.