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MONITORING CEPHEID PERIOD CHANGES FROM SAINT MARY'S UNIVERSITY

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Abstract

CCD observations in blue light with the 0.4-m telescope at Saint Mary's University are presented for the Cepheids SU Cyg, V402 Cyg, V1154 Cyg, V386 Cyg, V924 Cyg, MW Cyg, BB Her, GH Cyg, VY Cyg, TX Cyg, SZ Cyg, X Cyg, CD Cyg, and SV Vul. The data are used to establish new times of light maximum for the sample, and analyses of the new and previously-published O-C residuals are used to derive improved values for the rates of period change for each program object. The new values agree closely with predictions from stellar evolutionary theory for the rates at which intermediate-mass stars evolve through the Cepheid instability strip, and provide an excellent means of establishing the instability strip crossing mode for each variable.

1. Introduction

Classical Cepheid variables are valuable "standard candles" for measuring distances to nearby galaxies owing to their diagnostic brightness variations and the fact that their periods of pulsation are directly related to their luminosities via the Cepheid period-luminosity (PL) relation. All are post-main-sequence stars with masses of about 3–30$M_\odot$ that began their lives as hot hydrogen-burning main-sequence objects. They are now in a variety of instability strip crossing modes during advanced stages of nuclear fuel consumption: shell hydrogen burning (1st crossing), helium burning (2nd and 3rd crossings), and shell helium burning (4th and 5th crossings). The gradual changes that occur in their overall dimensions as they evolve can be inferred from photometric monitoring of their brightness variations. Such evolutionary changes—either increasing mean radius during evolution towards the cool side of the HR diagram (1st, 3rd, and 5th crossings) or decreasing mean radius during evolution towards the hot side of the HR diagram (2nd and 4th crossings)—are evident as either increases or decreases, respectively, in their periods of pulsation. The changes are not spectacular by any means, amounting in the most extreme cases to mere seconds or minutes per year in pulsation periods of days to months. The effects are cumulative, however, so the changes are significant and measurable as offsets of several hours or more in observed times for light maximum relative to those expected from observations made dozens of years previous.

2. Observations

The background and motivation for the present observational program of studying Cepheid period changes was described by Turner (1998). All observations described here were obtained using a Santa Barbara Instruments Group CCD camera (an ST-6 during 1996 and an ST-8 during 1997) on the 0.4-m telescope of the Burke-Gaffney Observatory at Saint Mary's University. Because of the bright sky background at the site and the lack of reliable guiding, the observations were made at a fairly low level of sophistication (see Turner 1998 for details). Cepheid fields were imaged at blue wavelengths—a compromise between the decreased sensitivity of the CCD and the lower brightness of Cepheids at short wavelengths and the larger light amplitudes of Cepheids in the blue—and brightness variations for program objects were derived by measuring magnitude differences relative to other stars, assumed light constant, in the same small fields of view. The reference stars ranged in brightness from about 9th to 15th magnitude in the blue, with as many as 1–8 being available for each star (the average was 4–5 stars per field). The reliability of the magnitude estimates varies according to the brightness of the Cepheid and the number of reference stars, but is typically in the range ±0.03 magnitude to ±0.06 magnitude.

Derived light curves for 14 of our program Cepheids are shown in Figure 1a–d relative to photoelectric light curves established previously by Szabados (1977, 1980, 1981) and Berdnikov (1992a, 1992b, 1992c, 1992d, 1992e). In some cases our observations sample the Cepheid light cycle reasonably completely (SU Cyg, V386 Cyg, V1154 Cyg, MW Cyg, BB Her, and CD Cyg); in other cases our observations miss important sections of the light curves (V402 Cyg, GH Cyg, VY Cyg, TX Cyg, and X Cyg). In all cases, however, it is possible to fit the existing observations to previously published light curves—which act as templates—to establish phase shifts for the program Cepheids relative to previous observations.

The technique used to match our new observations to existing data sets in both magnitude and phase space differs from the more sophisticated technique developed by Berdnikov (1992f) in that we use only observational data rather than a mean light curve for each Cepheid (see also Evans 1983). We initially matched the data by eye using identically-scaled versions of the light curves printed on transparencies, but found that computer matching of the data sets gave similar, although generally more precise, results, as well as a more direct means of establishing uncertainties for the estimates (our adopted uncertainties embrace an increase in photometric scatter of 10% about the minimum attained). Although Evans (1984) has pointed out that uncertainties in derived phase shifts as small as ±0.002 are readily attainable, in practice our algorithm generates uncertainties roughly twice as large. Given the nature of other factors that can influence O-C data—cyclical light travel time effects resulting from orbital motion (e.g., Szabados 1988), intrinsic period fluctuations, and other influences (see Evans 1984)—the observational uncertainties in O-C data tend to be small by comparison.

The newly-derived differences between Observed and Computed times of light maximum (O-C) relative to the ephemerides of Szabados (1977, 1980, 1981) are listed in Table 1 along with similar values obtained by applying the same technique to observations by Berdnikov. The plots of O-C differences compiled by Szabados and supplemented by more recent results (mostly from Szabados 1991) are shown in Figure 2. The temporal baselines in some cases exceed a century of observation.

Rates of period change for each program Cepheid were obtained by means of a least squares fit of a parabola to the O-C data, a technique described in some detail by Belserene (1986). Since the O-C data compiled by Szabados include assigned weights based upon a quality assessment of each measured time of light maximum, the parabolic fits included weights for each data point. Szabados generally assigned weights of 0.5 for visual observations, 1.0 for photographic observations, and 3.0 for photoelectric observations. Weights of 2.0 were assigned infrequently, but seem to be appropriate for the quality of our new estimates from CCD imaging (as indicated in Table 1). The results obtained for each Cepheid are summarized in Table 2 and are illustrated in Figures 3 and 4. The temporal O-C changes for each Cepheid need not be strictly parabolic, as pointed out by Fernie (1990), but generally such relations provide the simplest and most informative match to the data. As we point out here, the results strongly support the belief that O-C variations are dominated by evolutionary effects.

Table 1. Newly derived O-C residuals for program Cepheids.

Cepheid	*JD$_{max}$*	*E*	*O-C (days)*	*Weight*	*Data Source*
X Cyg	2447762.863	+240	-0.244 ±0.049	3	Berdnikov (1992d)
	2448106.861	+261	-0.359 ±0.049	3	Berdnikov (1992e)
	2450253.618	+392	-0.211 ±0.049	2	This Paper
SU Cyg	2450262.102	+2206	+0.012 ±0.015	2	This Paper
SZ Cyg	2447417.080	+242	+0.418 ±0.076	3	Berdnikov (1992c)
	2450272.980	+431	+0.596 ±0.091	2	This Paper
TX Cyg	2447428.430	+247	+1.284 ±0.066	3	Berdnikov (1992c)
	2450650.202	+466	+1.970 ±0.191	2	This Paper
VY Cyg	2446620.255	+455	+0.046 ±0.055	3	Berdnikov (1992a)
	2447413.884	+556	+0.120 ±0.039	3	Berdnikov (1992c)
	2450273.927	+920	+0.222 ±0.063	2	This Paper
CD Cyg	2450268.492	+377	+0.439 ±0.060	2	This Paper
GH Cyg	2446621.539	+496	+0.103 ±0.023	3	Berdnikov (1992a)
	2450256.908	+961	+0.134 ±0.039	2	This Paper
MW Cyg	2446621.810	+621	+0.173 ±0.018	3	Berdnikov (1992a)
	2447419.754	+755	+0.203 ±0.018	3	Berdnikov (1992c)
	2447753.181	+811	+0.173 ±0.018	3	Berdnikov (1992d)
	2450295.736	+1238	+0.120 ±0.030	2	This Paper
V386 Cyg	2450311.467	+1433	+0.200 ±0.016	2	This Paper
	2450642.586	+1496	+0.089 ±0.016	2	This Paper
V402 Cyg	2446622.172	+1128	+0.002 ±0.022	3	Berdnikov (1992a)
	2450284.263	+1967	-0.004 ±0.039	2	This Paper
V924 Cyg	2447417.167	+781	-0.228 ±0.017	3	Berdnikov (1992c)
	2450264.225	+1292	-0.192 ±0.028	2	This Paper
V1154 Cyg	2447424.785	+1204	+0.089 ±0.015	3	Berdnikov (1992c)
	2447799.110	+1280	+0.079 ±0.015	3	Berdnikov (1992d)
	2450266.736	+1781	+0.050 ±0.022	2	This Paper
BB Her	2446620.961	+525	+0.001 ±0.023	3	Berdnikov (1992a)
	2450217.334	+1004	+0.069 ±0.038	2	This Paper
SV Vul	2446598.449	+64	+3.463 ±0.135	3	Berdnikov (1992a)
	2447093.526	+75	+3.598 ±0.112	3	Berdnikov (1992b)
	2447408.535	+82	+3.643 ±0.135	3	Berdnikov (1992c)
	2447768.268	+90	+3.418 ±0.180	3	Berdnikov (1992d)
	2450285.456	+146	+0.898 ±0.315	2	This Paper

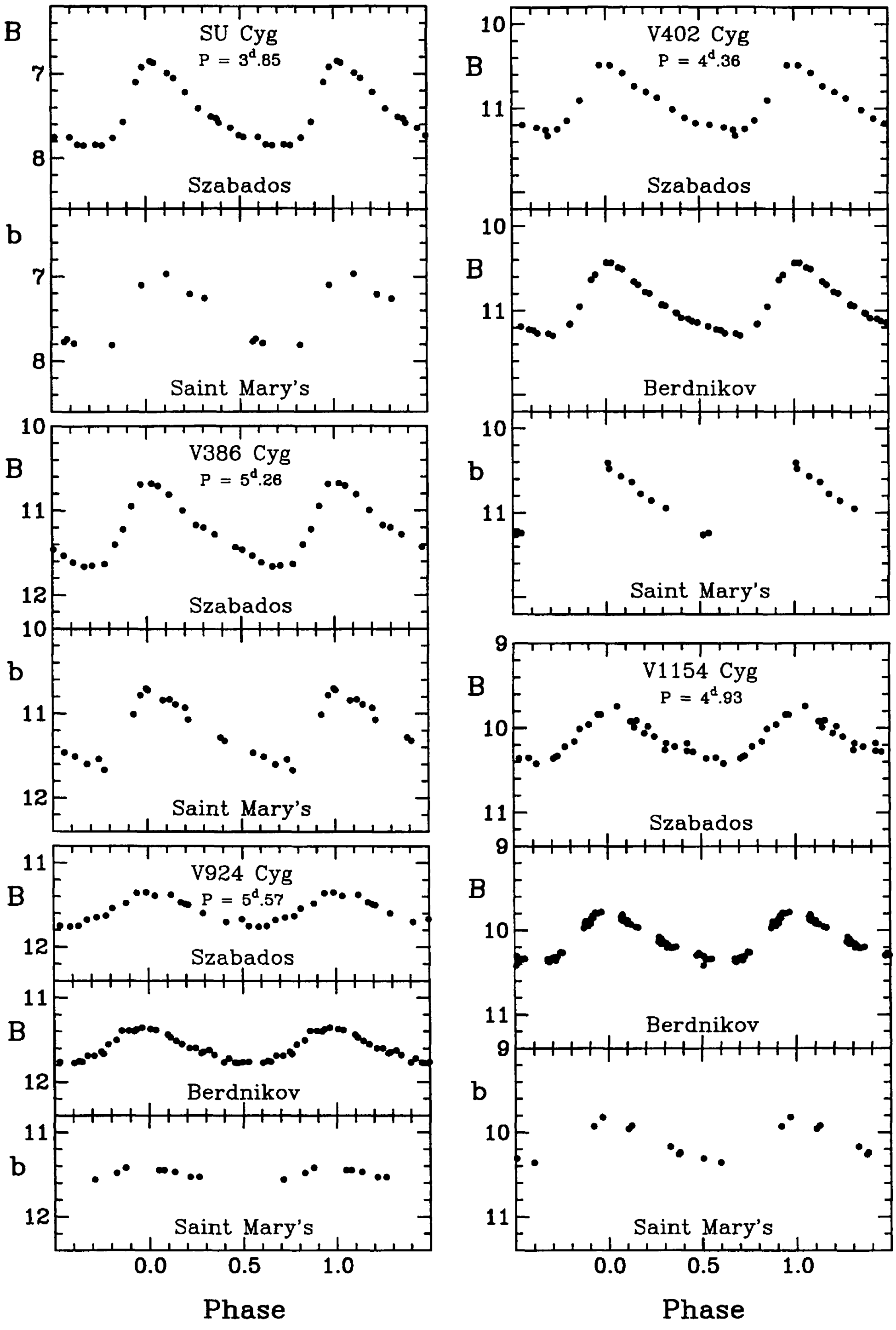

Figure 1a–d shows a montage of light curves for the fourteen Cepheids observed in the program. In each section the top diagram illustrates the B-magnitude light curve obtained by Szabados (1977, 1980, 1981) and the bottom diagram the instrumental b-magnitude light curve obtained from our CCD observations normalized to the former. B-magnitude observations by Berdnikov (1992a, 1992b, 1992c, 1992d, 1992e) are illustrated in the center diagram of each section, where available. Phases are relative to the ephemerides of Szabados. Figure 1a above shows light curves obtained by Szabados and Saint Mary's for SU Cyg and V386 Cyg, and light curves obtained by Szabados, Berdnikov, and Saint Mary's for V924 Cyg, V402 Cyg, and V1154 Cyg.

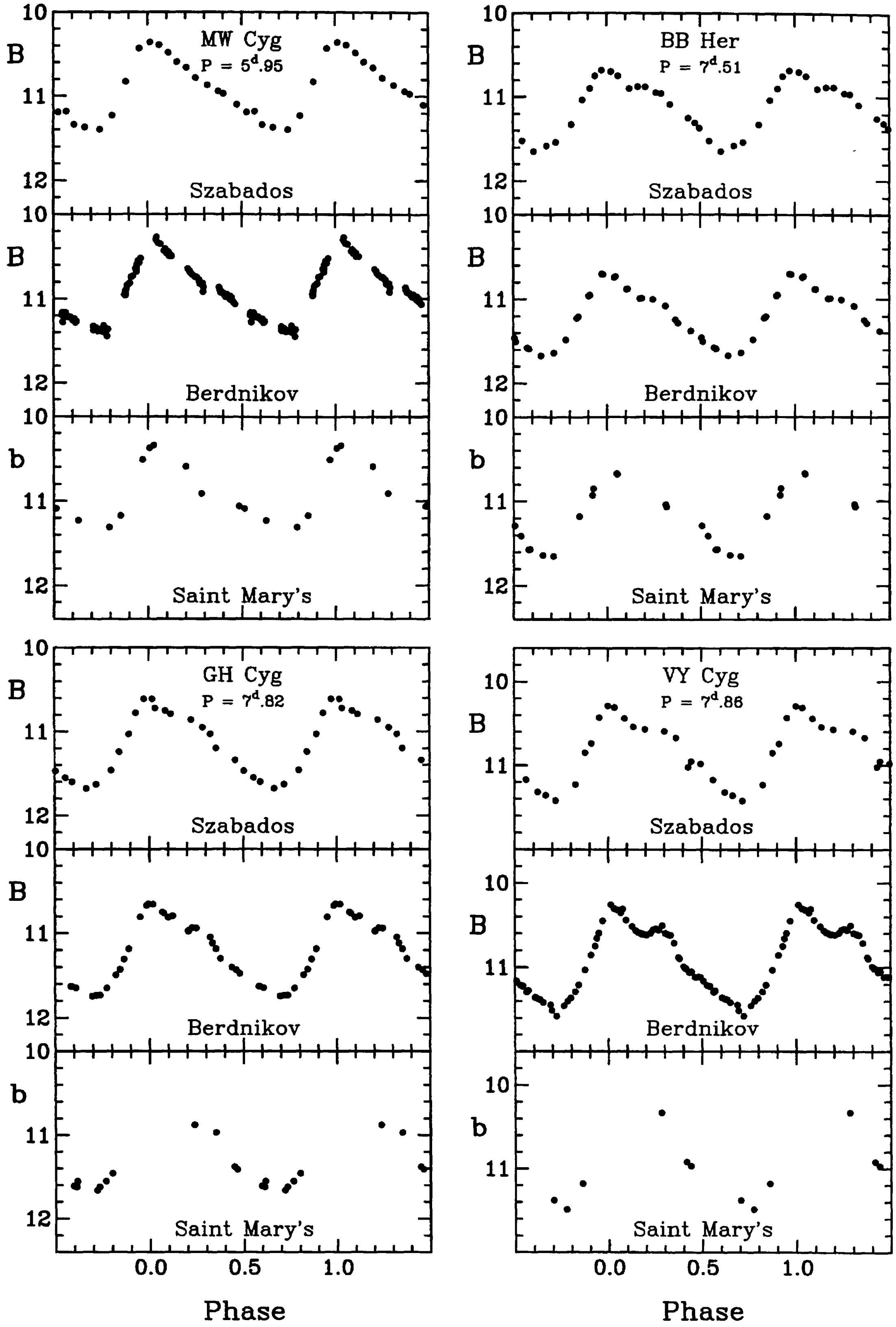

Figure 1b. Light curves obtained by Szabados, Berdnikov, and Saint Mary's for MW Cyg, BB Her, GH Cyg, and VY Cyg. See caption for Figure 1a for full explanation.

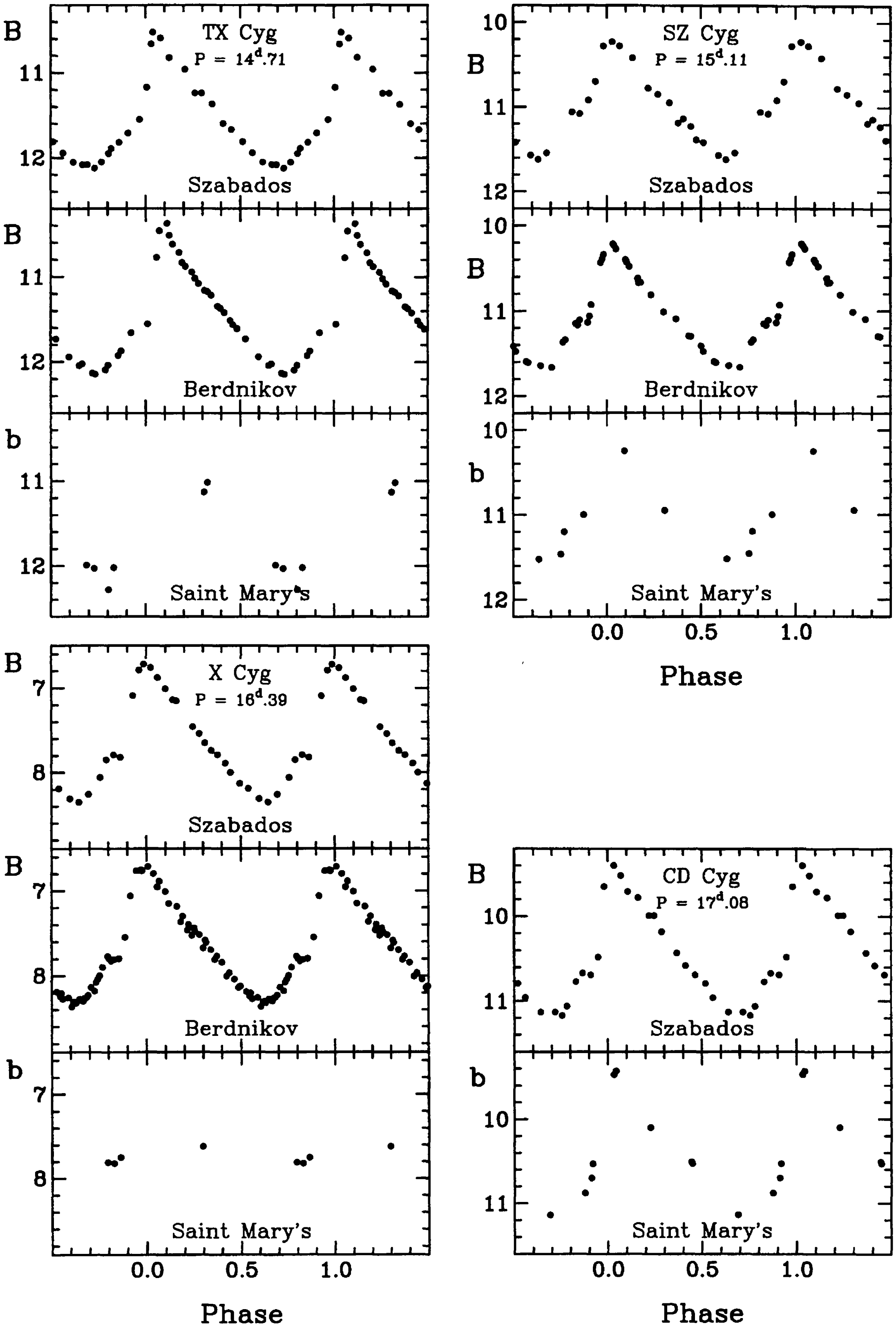

Figure 1c. Light curves obtained by Szabados, Berdnikov, and Saint Mary's for TX Cyg, SZ Cyg, and X Cyg, and light curves obtained by Szabados and Saint Mary's for CD Cyg. See caption for Figure 1a for full explanation.

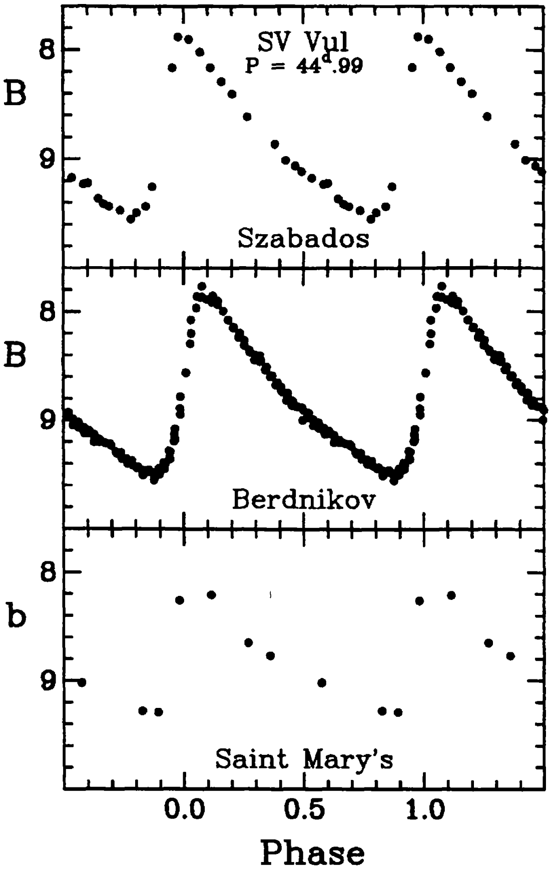

Figure 1d. Light curves obtained by Szabados, Berdnikov, and Saint Mary's for SV Vul. See caption for Figure 1a for full explanation.

3. Results for individual Cepheids

We discuss here results for individual program objects, some of which exhibit interesting effects either in their light curves or in the nature of their period changes. The Cepheids are discussed in order of increasing pulsational period.

3.1. SU Cyg

Although this short-period Cepheid has two blue unresolved companions (Evans and Bolton 1987, 1990), they appear to have little effect on the star's light curve. Nevertheless, the spectroscopic binary nature of the star (Evans 1988) is of some importance, owing to the large mass of the system and the fact that the orbital plane is oriented nearly in the line of sight. The orbital period of the B-star subsystem relative to the Cepheid is 1½ years; the excursions of SU Cyg about the system barycenter amount to light travel time delays of roughly ±0.008 day, or ±0.002 in phase. The latter is typical of the best attainable accuracy in derived phase shifts (Evans 1984), and may account for some of the systematic scatter in the star's O-C residuals. Our new photometry agrees well with the data of Szabados (Figure 1a), with a phase shift (Table 1) consistent with the long-term trend evident in Figure 2a, which includes updated O-C estimates for SU Cyg from Szabados (1991). We have derived the rate of period change for the Cepheid using only data for times of maximum lying within roughly ±50 days of nodal passage, in order to insure minimal influence from light travel time delays. The derived rate of period increase (Table 2) is a close match to what is predicted theoretically for a star in the 3rd crossing of the instability strip (Figure 3), although, given the restricted nature of the solution, additional observations might be useful. Note that our 1996 observations were made between times of nodal passage for the system, which may partly account for the deviation of our O-C datum from the predicted trend.

3.2. V402 Cyg

Our light curve for this Cepheid (Figure 1a) lacks observations on the rising light portion, yet that does not affect the derivation of the phase shift (Table 1). The O-C variations for the Cepheid are very small (Figure 2a), and the derived rate of period increase (Table 2) is smaller than its estimated uncertainty. The data are weakly consistent with a Cepheid in the 3rd crossing of the instability strip (Figure 3), but the uncertainty in the result also permits a negative rate of period change that is more consistent with a 2nd crossing (Figure 4). A longer baseline of observations is needed to decide between the two possibilities.

3.3. V1154 Cyg

Our light curve for this Cepheid (Figure 1a) is fairly complete, and the derived phase shift (Table 1) is consistent with other recent determinations. Owing to the large scatter in older O-C estimates for the star (Figure 2a), the derived rate of period increase (Table 2) has a large uncertainty that embraces the possibility of a decreasing period (Figures 3 and 4). Examination of the two possibilities indicates close consistency with a 3rd crossing of the instability strip, our preferred option.

3.4. V386 Cyg

Our light curve for this Cepheid (Figure 1a) has excellent phase coverage and provides estimates of phase shift for two seasons (Table 1). Updated O-C estimates are provided by Szabados (1991). The star lacks O-C data from the early years of the century (Figure 2a), but the derived rate of period increase (Table 2) is nevertheless a close match to what is predicted theoretically for a star in the 3rd crossing of the instability strip (Figure 3).

3.5. V924 Cyg

This small-amplitude Cepheid has a sinusoidal light curve (Figure 1a). Our observations in the 1996 season seem to imply a decrease in light amplitude, but we are planning further observations in order to explore other possibilities—for example, that the effect originates in the photometric properties of our CCD device. Despite the star's low light amplitude, a good match is obtained with published light curves, with allowance for a reasonable phase shift (Table 1). Updated O-C estimates are provided by Szabados (1991). The star does not have a long history of O-C observations (Figure 2a), but a large rate of period change is indicated. The rate of period increase is too large for a 3rd crossing of the instability strip (Figure 3), but is reasonably consistent with a 5th crossing, for which our theoretically-predicted relation is less well-established. Given that some small-amplitude Cepheids with sinusoidal light curves are suspected to be in the 1st crossing (see Turner 1998), that is also a possibility that needs to be explored. We encourage others to include the star in their observing programs.

3.6. MW Cyg

Our light curve for this Cepheid (Figure 1b) has excellent phase coverage, and provides a good estimate of phase shift (Table 1). The O-C data (Figure 2a) include the updated O-C estimates of Szabados (1991), and indicate a derived rate of period increase (Table 2) that is a close match to what is predicted theoretically for a star in the 3rd crossing of the instability strip (Figure 3). The star is also a recognized spectroscopic binary (Gorynya *et al.* 1992).

3.7. BB Her

The light curve (Figure 1b) and O-C data (Table 1, Figure 2a) for this Cepheid were discussed earlier (Turner 1998), and we reiterate here that the derived rate of period increase (Table 2) is a close match to what is predicted theoretically for a star in the 3rd crossing of the instability strip (Figure 3).

3.8. GH Cyg

Our observations for this Cepheid (Figure 1b) miss light maximum, but catch the bump on the declining light portion of the light curve. The derived phase shift (Table 1) is consistent with other recent values, but the overall O-C data over the last century (Figure 2a) exhibit little variation. The derived small rate of period increase is consistent with a Cepheid in the 3rd crossing of the instability strip (Figure 3), but the uncertainty in the result also permits a negative rate of period change that is equally consistent with a 2nd crossing (Figure 4). Once again it appears that a longer baseline of observations is needed to decide between the two possibilities.

3.9. VY Cyg

Our observations for this Cepheid (Figure 1b), like those for GH Cyg, miss light maximum but catch the bump on the declining light portion of the light curve. The derived phase shift (Table 1) is consistent with the long-term trend (Figure 2b), and the Cepheid has a well defined rate of period increase (Table 2) consistent with a 3rd crossing of the instability strip (Figure 3).

3.10. TX Cyg

Our scattered observations for this Cepheid (Figure 1c) yield a phase shift (Table 1) of low reliability. Further observations are planned. The O-C data (Figure 2b) include the updated O-C estimates of Szabados (1991), and indicate a rapid rate of period increase (Table 2). The results are consistent with a Cepheid in the 3rd crossing of the instability strip (Figure 3).

3.11. SZ Cyg

Our small number of observations for this Cepheid (Figure 1c) sample the light curve reasonably well, and yield a phase shift (Table 1) consistent with other recent estimates. The scattered O-C data (Figure 2b) include the updated O-C estimates of Szabados (1991), and yield a rate of period increase (Table 2) that is less well determined than that for TX Cyg, but still consistent with a Cepheid in the 3rd crossing of the instability strip (Figure 3). The Cepheid is suspected to be a binary on the basis of the sinusoidal trends in its O-C data (Szabados 1981), a possibility that might also account for the small deviation of the star's calculated rate of period change from the theoretical relation. A longer temporal baseline of data could improve the quality of the star's observed rate of period change.

3.12. X Cyg

By chance our four observations of this Cepheid (Figure 1c) were all near median light, yet they do bracket light maximum and catch the bump on the rising light portion of the light curve. Normally such a small number of observations with such a pathological distribution would be insufficient for studying a star's period change, but for X Cyg they happen to yield a reasonably precise phase shift (Table 2) that is consistent with recent trends. The O-C data for X Cyg (Figure 2b) include the updated O-C estimates of Szabados (1991), and exhibit cyclical patterns that are not fully explained. Continued observations of the star are needed to extend the baseline of data. The derived rate of period increase (Table 2) is consistent with a Cepheid in the 3rd crossing of the instability strip (Figure 3), and the slight deviation from the theoretical relation is probably caused by the unbalanced nature of the Cepheid's superimposed cyclical variations.

3.13. CD Cyg

Our observations for this Cepheid (Figure 1c) provide fairly complete phase coverage and a good estimate for the phase shift (Table 1). The O-C data (Figure 2b) yield a rate of period increase (Table 2) that is a close match to predictions for a 3rd crossing of the instability strip (Figure 3).

3.14. SV Vul

Our observations for this Cepheid (Figure 1d) hint at a smaller light amplitude than indicated by the photoelectric data. We hesitate to draw such a conclusion, however, since there are very few comparison stars in the CCD field and all are rather faint. The observations yield a reasonable phase shift (Table 1), and the O-C data (Figure 2b) confirm the well-established period decrease for the star (Table 2). Included in Figure 2b are data from Berdnikov (1994) and Szabados (1991) adjusted to the system defined by Szabados (1981). There are cyclical variations for SV Vul like those for X Cyg, but they seem to be averaged out over the long temporal baseline. The derived rate of period decrease is a close match to predictions for a 2nd crossing of the instability strip (Figure 4).

Table 2. Derived rates of period change for program Cepheids.

Cepheid	*Period (days)*	*dP/dt (s/yr)*		*log (dP/dt)*	*Level of Significance*	*Inferred Instability Strip Crossing*
SU Cyg	3.846	+0.032	±0.021	-1.500	Fair	3rd
V402 Cyg	4.365	+0.004	±0.024	-2.428	Low	3rd or 2nd
V1154 Cyg	4.925	+0.079	±0.094	-1.103	Low	3rd [1]
V386 Cyg	5.258	+0.152	±0.046	-0.818	High	3rd
V924 Cyg	5.571	+1.660	±0.307	+0.220	High	5th
MW Cyg	5.955	+0.171	±0.077	-0.767	Fair	3rd
BB Her	7.508	+0.378	±0.077	-0.423	High	3rd
GH Cyg	7.818	+0.028	±0.116	-1.555	Low	3rd (or 2nd)
VY Cyg	7.857	+0.185	±0.047	-0.733	High	3rd
TX Cyg	14.709	+3.196	±0.213	+0.505	High	3rd
SZ Cyg	15.110	+0.947	±0.235	-0.024	High	3rd
X Cyg	16.386	+1.460	±0.267	+0.164	High	3rd
CD Cyg	17.072	+5.223	±0.458	+0.718	High	3rd
SV Vul	45.062	-214.281	±5.472	(+2.331)[2]	High	2nd

NOTES

[1] *The data also permit the star to be in the 2nd crossing, but that seems less likely.*

[2] *Brackets indicate that log (-dP/dt) is listed.*

4. Evolutionary effects on Cepheid periods

A striking feature of Figures 3 and 4 is the close agreement of the data with the theoretically-predicted relations based upon evolution through the instability strip. The agreement between observation and theory is excellent, particularly when uncertainties in the least squares O-C fits are taken into account. A comparison with similar diagrams given by Turner (1998) for a larger and more heterogeneous sample of Cepheids provides convincing evidence for the value of continued monitoring of Cepheids for purposes of establishing their likely instability strip crossing modes. The techniques used to establish light curves and times of light maximum for Cepheids from photoelectric and CCD observations provide reasonably reliable results; that is evident in the smaller scatter present in recent O-C data (see Figure 2). As longer temporal baselines are established, the precision and accuracy of least squares fits to establish rates of period change must certainly improve. Despite the low level of sophistication of our observational campaign, the results prove to be adequate for improving the known rates of period change for our program stars. For all but two of the fourteen Cepheids observed, the results are sufficient to establish the likely instability strip crossing mode of the star with some confidence. In fact, the close coincidence between observation and theory—for nine of the Cepheids the agreement is exact, within the quoted uncertainties—attests to the general validity of our theoretical understanding of the evolution of intermediate mass stars through the instability strip. Future observations of a larger sample may even improve the situation.

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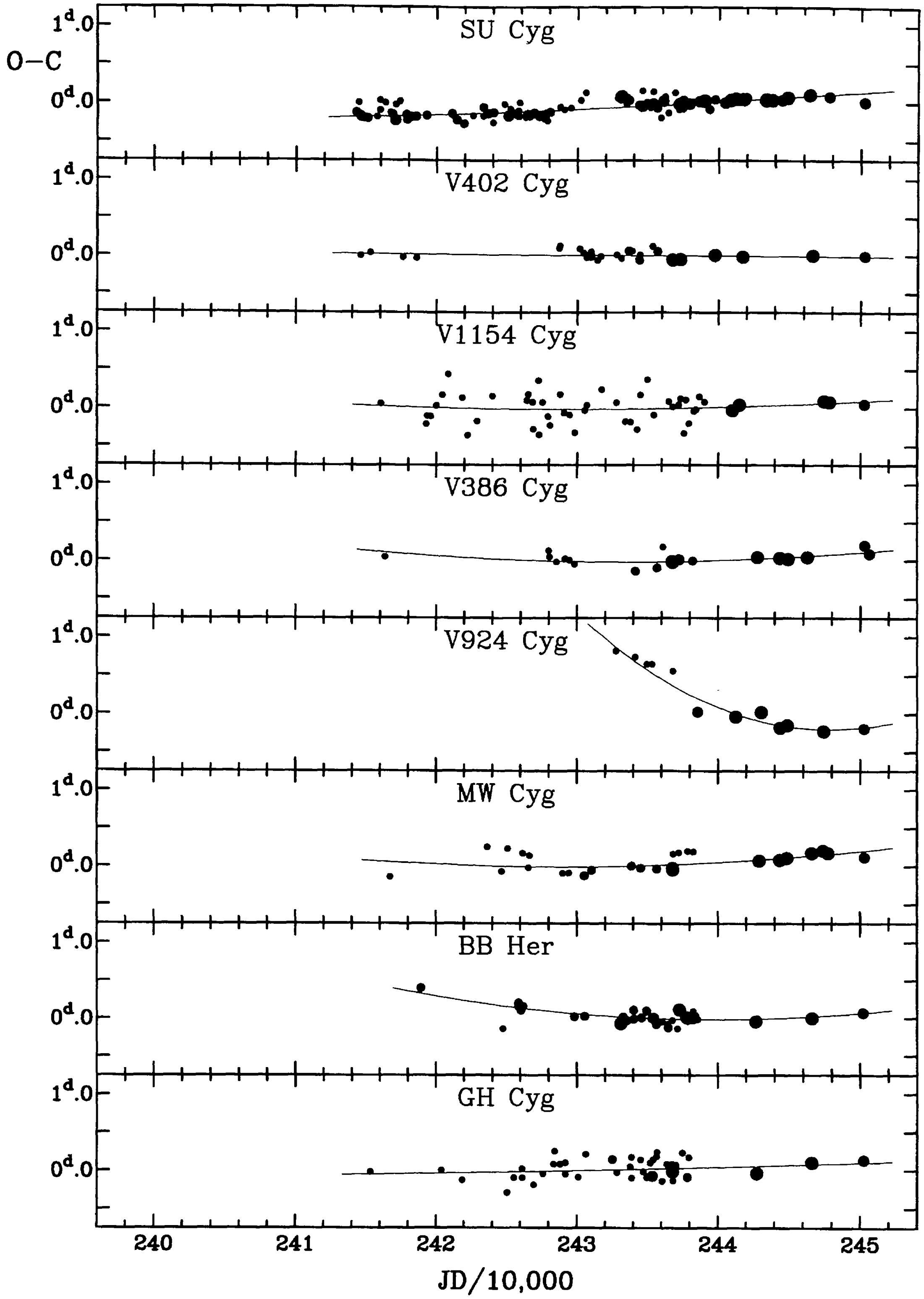

Figure 2a–c shows a montage of O-C diagrams for the fourteen Cepheids observed in the program. The size of the symbol is proportional to the weight assigned to the data point by Szabados or in this paper (for recent observations). The lines are parabolic fits that have been made to the data by means of weighted least-squares analyses. Figure 2a above shows the O-C diagrams for SU Cyg, V402 Cyg, V1154 Cyg, V386 Cyg, V924 Cyg, MW Cyg, BB Her, and GH Cyg.

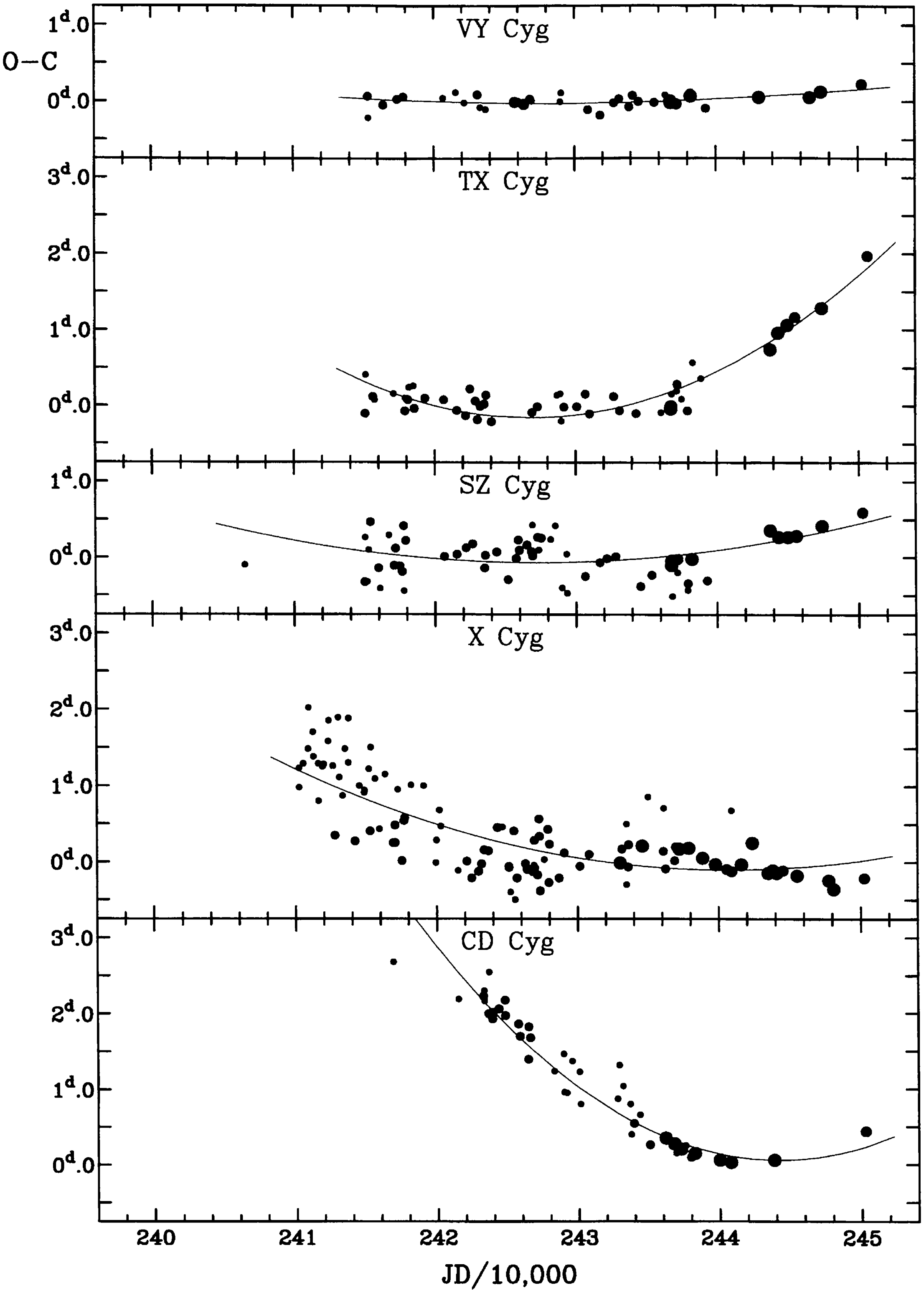

Figure 2b. O-C diagrams for VY Cyg, TX Cyg, SZ Cyg, X Cyg, and CD Cyg. See caption for Figure 2a for full explanation.

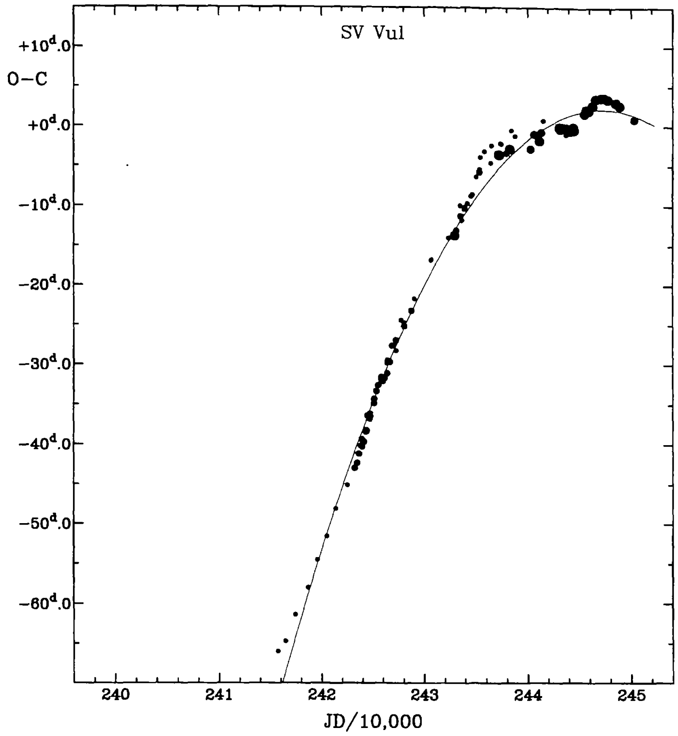

Figure 2c. O-C diagram for SV Vul. See caption for Figure 2a for full explanation.

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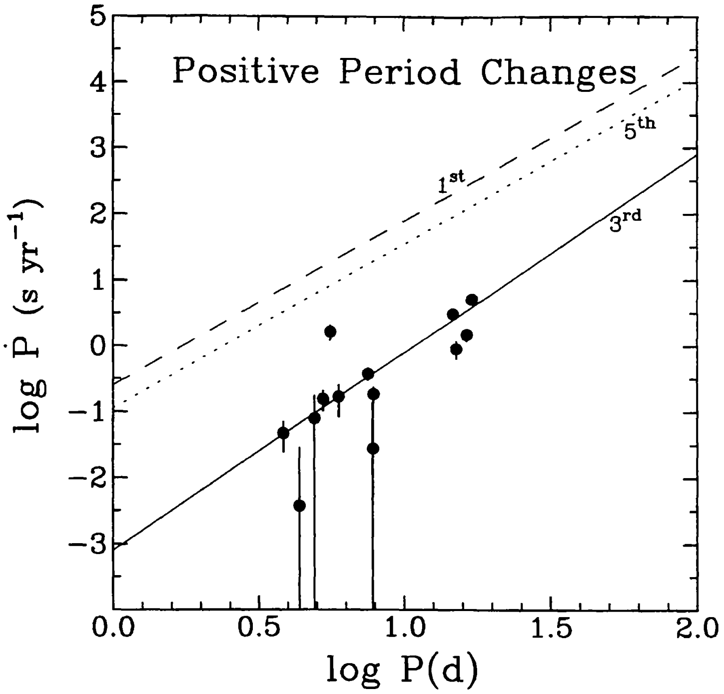

Figure 3. The dependence on period of the rates of period change for Cepheids in the program with increasing periods. Each data point is plotted with its associated uncertainty, where that is larger than the size of the symbol. Note that for three of the Cepheids the uncertainties go off scale at the bottom of the diagram. The different lines represent the predictions for 1st, 3rd, and 5th crossings of the instability strip.

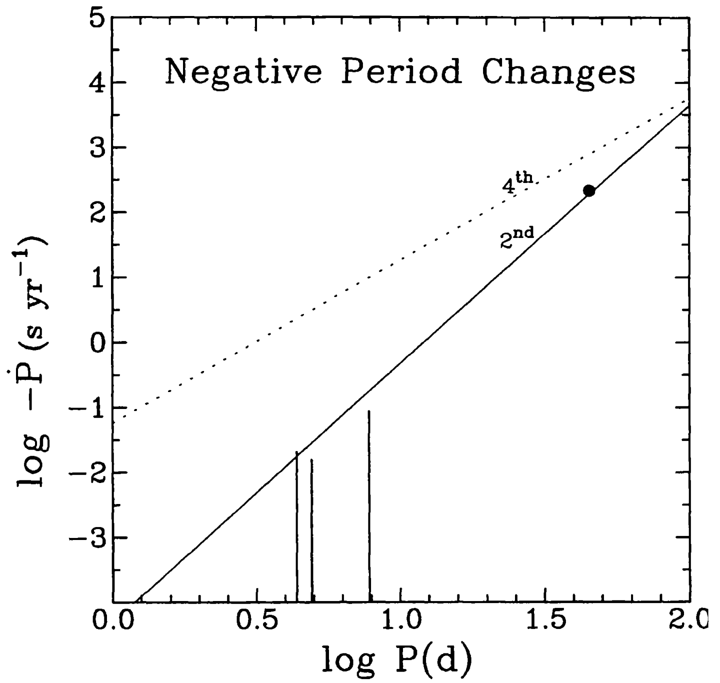

Figure 4. The dependence on period of the rates of period change for Cepheids in the program with decreasing periods. Symbols are the same as in Figure 3. Note that three of the Cepheids have positive rates of period change (they are plotted in Figure 3), but their associated uncertainties include the rates of negative period change shown here. The different lines represent the predictions for 2nd and 4th crossings of the instability strip.