INFLATION OF AAVSO SUNSPOT COUNTS

Grant Foster
AAVSO Headquarters
25 Birch Street
Cambridge, MA 02138

Abstract

I consider the process of continually recomputing the k coefficients used in the AAVSO sunspot counts, and whether they may be subject to systematic inflation.

1. Introduction

Recently, Schaefer (1997) has raised the possibility that the method of estimating sunspot counts based on AAVSO visual estimates may contain a hitherto-undetected flaw: that the process of occasionally recomputing the k coefficients inherently causes a small inflation each time they are computed. The k coefficients are introduced because observers tend to report only a fraction of the sunspots actually present; we compensate by estimating a factor k_j for each observer j such that on average, the real count R is just k_j times the observer's reported count c_j

$$R = k_j c_j \tag{1}$$

Of course, it is nontrivial to estimate the correct coefficients k_j for each observer. They are periodically recomputed by comparing each observer's reported counts to the overall AAVSO estimated sunspot numbers for a given time period. Schaefer suggests that the method used for this recomputation will inherently inflate the estimated counts each time it is done, for two reasons: first, because of differences between the "true" k values and our estimates of them; second, due to random errors in observers' reported counts. He goes on to suggest an alternative procedure which does not suffer from inflation.

If true, then the AAVSO sunspot numbers will also exhibit a small but steady inflation. Schaefer estimates this inflation to be 0.3% each time the k values are re-computed. He further estimates that this has been done 18 times since AAVSO began computing estimated sunpot numbers; this implies a total inflation of $1.003^{18} = 1.0554$, or about a 6% increase. Such an increase is of the same order of magnitude as the inherent errors in the counts themselves, so it may reduce their scientific usefulness, but certainly does not invalidate them. Of course, any cumulative inflation will introduce a spurious trend in our measurement of solar activity; clearly any method should be revised which suffers from systematic inflation.

Therefore I investigate the present method for computing the AAVSO sunpot counts, and of revising the k values. Using a different error model than Schaefer, I find that the process *does* suffer inflation due to random errors in observers' reported values.

2. Basic Variables

Consider the observations of sunpots over a period of N days t_a, $a = 1, 2, ..., N$. Let the actual sunspot number on day t_a be R_a. However, any given observer will not report this number, due to two effects: first, observers tend to report only a fraction of the spots actually present; second, any real observation carries with it some random error, however small.

Let J be the number of observers reporting. Then each observer $j = 1, 2, ..., J$ reports a certain *count* c_{ja} for a given day t_a. Hence for each observer we define a constant k_j such that *on average*, the reported count is the fraction k_j^{-1} of the real count R_a, i.e.,

$$\langle c_{ja} \rangle = k_j^{-1} R_a, \tag{2}$$

where angle brackets "$\langle\rangle$" denote the expectation value of a random variable.

A. H. Shapley (1949) noted that the magnitude of the *random* error in c_{ja} is larger when the sunspot count is larger; it seems not unreasonable to assume that it is proportional to R_a. Then the random errors in c_{ja} can be written as

$$c_{ja} = k_j^{-1} R_a (1 + \epsilon_{ja}) \tag{3}$$

where the variance of any given random error ϵ_{ja} is assumed independent of the sunspot count, depending only on the observer

$$\langle \epsilon_{ja}^2 \rangle = \sigma_j^2. \tag{4}$$

For equation (2) to hold, the expected value of any given random error is

$$\langle \epsilon_{ja} \rangle = 0. \tag{5}$$

This is the error model on which we base our estimation of the sunspot count from different observers.

I emphasize that the factors k_j are not random variables; by definition, they are constants defined by equation (2). The random part of any observation is encoded in the random error ϵ_{ja}.

Of course, we don't know the actual values of the constants k_j giving the ratio of true to observed counts; instead we use estimates $\hat{k}_j$. Given the observed counts c_{ja} and the *estimated* factors $\hat{k}_j$, we can compute the overall AAVSO estimated sunspot count $\hat{R}_a$ for a given day t_a as the average for all observers

$$\hat{R}_a = \frac{1}{J} \sum_{j=1}^{J} \hat{k}_j c_{ja}. \tag{6}$$

Actually, the AAVSO sunspot count is a *weighted* average, and includes only those observers who actually report data for the day t_a, but it can be shown without too much difficulty that these differences will not affect the following arguments.

3. Recomputing the k coefficients

Periodically, the k_j coefficients are recomputed. At present, the *new* estimates $\tilde{k}_j$ are given by

$$\tilde{k}_j = 10^{[N^{-1}\sum_{a=1}^{N}\log(\hat{R}_a/c_{ja})]}. \tag{7}$$

I can rewrite this as

$$\tilde{k}_j = [\prod_{a=1}^{N}\frac{\hat{R}_a}{c_{ja}}]^{1/N}, \tag{8}$$

i.e., the new estimate is the geometric mean of all available ratios $\hat{R}_a/c_{ja}$.

Substituting equations (3) and (6),

$$\tilde{k}_j = [\prod_{a=1}^{N}\frac{J^{-1}\sum_{p=1}^{J}\hat{k}_p c_{pa}}{c_{ja}}]^{1/N} = [\prod_{a=1}^{N}\frac{J^{-1}\sum_{p=1}^{J}\hat{k}_p k_p^{-1} R_a(1+\epsilon_{pa})}{k_j^{-1}R_a(1+\epsilon_{ja})}]^{1/N}$$

$$= J^{-1}k_j\,[\prod_{a=1}^{N}\sum_{p=1}^{J}\hat{k}_p k_p^{-1}\frac{1+\epsilon_{pa}}{1+\epsilon_{ja}}]^{1/N}. \tag{9}$$

3.1. Inflation

Now, if there are no random daily errors, so that all $\epsilon_{ja} = 0$, then (9) becomes

$$\tilde{k}_j = k_j J^{-1}\sum_{p=1}^{J}\hat{k}_p k_p^{-1}. \tag{10}$$

We have the remarkable result that the new estimates $\tilde{k}_j$ do not depend at all on the specific values of our original estimates $\hat{k}_j$, except for a multiplicative constant which is the average ratio of assumed to true $\hat{k}_j/k_j$ from the original estimates. In fact our new estimates give *exactly the correct values*, with a scale factor such that the *average* ratio of our new estimates to the true values becomes

$$J^{-1}\sum_{j=1}^{J}\tilde{k}_j/k_j = J^{-1}\sum_{p=1}^{J}\hat{k}_j/k_j. \tag{11}$$

We see that the average ratio of estimated to true k values remains constant, so there is *no inflation* due to differences between the true coefficients k_j and our estimates of them $\hat{k}_j$.

Of course, the k coefficients change over time. This can be due to aging of the optics (the eye), the accumulation of greater experience and therefore better technique, the acquisition of better equipment, etc. However, these changes are by no means random, and as shown in equation (10), the process of recomputation does not inflate the k values, it merely corrects them.

Now consider the case *with* random daily errors, i.e., the ϵ_{ja} are not all zero, but obey equations (4) and (5). Consider also the simplified case in which all our original estimates $\hat{k}_j$ are exactly correct, so that all $\hat{k}_p/k_p = 1$. Then we can *approximate* the expected value of (9) as (see appendix)

$$\langle \tilde{k}_j \rangle \approx k_j(1 + \sigma_j^2). \tag{12}$$

Therefore, in spite of the fact that our initial assumed values $\hat{k}_j$ were *exactly correct*, the new estimates $\tilde{k}_j$ will, on average, be higher by a factor $1 + \sigma_j^2$. This *is* inflation. In fact, if we *iterate* this process, using the new estimates to generate even newer estimates, these will be inflated further; they will, on average, increase by a factor

$$\gamma = 1 + \sigma^2 = J^{-1} \sum_{j=1}^{J} (1 + \sigma_j^2), \tag{13}$$

each time we recompute the k factors.

If the RMS random daily error averaged over all observers σ is 0.0548 (i.e., about 5.5% random scatter in observers' counts – not an unreasonable estimate), then the factor γ will be 1.003, the value estimated by Schaefer.

4. Comparison of AAVSO to Zurich sunspots numbers

We are not limited to theoretical considerations of possible inflation of AAVSO sunspot counts; we can look for inflation by comparing AAVSO counts to the Zurich sunspot numbers. Figure 1 shows AAVSO and Zurich mean monthly sunspot counts from 1945 to 1994. Visual inspection indicates that they are nearly identical; certainly they reveal the same general changes in solar activity. Yet the scale of this plot is so large as to mask small differences between AAVSO and Zurich counts. It is much more revealing (and to the point) to examine the ratio of AAVSO to Zurich counts.

These ratios show very large fluctuations when the count gets very small, so I will consider *only* those months with a mean Zurich count ≥ 20. If there is inflation according to (12), then the AAVSO count will grow exponentially, while its logarithm will grow linearly; therefore I consider the values $\log(R_A/R_Z)$, where R_A is the AAVSO count and R_Z the Zurich count; these are plotted in Figure 2.

First of all, from 1945 to 1951 the AAVSO count rose dramatically relative to the Zurich count, increasing 5.3% per year for a total inflation of 36%! This clear divergence between the two indices motivated a revision of the AAVSO k coefficients in 1951; the factors were re-calibrated to bring the AAVSO counts to the same scale as the Zurich counts. This is the cause of the sudden jump in R_A/R_Z at the beginning of 1951.

There are at least two interpretations of the behavior from 1951 to the present. At first glance it indicates two episodes during which the ratio R_A/R_Z evolves steadily. From 1951 to 1967, the AAVSO counts show a steady *deflation* relative to Zurich! R_A/R_Z decreased 1.3% per year for a total *de*flation of 18%. Then, from 1967 to 1994, there has been a steady increase at 0.2% per year for a total inflation of 5.9%. It is also possible that the AAVSO numbers inflated steadily from 1951 to 1994, but that there *seems* to be a deflation from 1951 to 1967 because the AAVSO numbers are artificially low from 1962 to 1967. In fact, removing the counts from 1962 to 1967 allows a nice straight-line fit to the remainder of the data from 1951 on (Figure 3), indicating an increase of R_A/R_Z by 0.26% per year for a total inflation of 12% from 1951 to 1994.

In any case, the evidence points to a clear inflation of R_A relative to R_Z since 1967. This is further indication that the theoretical basis for inflation is sound, and that AAVSO should revise its procedure for recomputing the k factors.

5. Recommendations

Schaefer suggests a revision which eliminates inflation in the AAVSO procedure: instead of estimating the sunspot number as the average (equation 6), use the logarithmic average

$$\log \hat{R}_a = \frac{1}{J} \sum_{j=1}^{J} \log(\hat{k}_j c_{ja}), \qquad (14)$$

and use *these* in the present procedure for revising the k_j coefficients. This procedure appears to be free from any systematic inflation. However, I point out that the original procedure seemed to make perfect sense at the time; only careful consideration uncovered its flaw. Therefore I recommend that some time be taken, and thought given, to settle on a revised procedure for computing AAVSO sunspot numbers. I further recommend that this be done soon (certainly before the next re-computation of the k_j coefficients); further delay will only exacerbate the problem of inflated AAVSO sunspot numbers. Unless a superior method is suggested, or a flaw is uncovered, Dr. Schaefer's suggestion seems both simple and correct.

It also seems advisable that, for a time at least, the AAVSO sunspot counts and k_j coefficients should be computed by *two* methods, the old way and the new. These estimates should be published side by side for several years at least. This will provide continuity with older data, and will serve as a useful check on the behavior of both methods.

The tricky problem, of course, is to revise the existing AAVSO sunspot counts from 1945 to the present. It appears that the original, raw data are no longer available for analysis; therefore it will not be possible to reconstruct the historical sunspot counts using a revised procedure. This leaves a number of possibilities, including: 1, leave the AAVSO numbers alone, but publish stern warnings that a false inflationary trend is believed to be present; 2, de-trend the AAVSO numbers by a constant factor per year, from 1951 to the present.

6. Additional Comment

Another effect appears in the comparison of AAVSO to Zurich sunspot numbers. There seems to be a cyclic fluctuation in the ratio R_A/R_Z, which follows the solar cycle; the ratio reaches a peak some time *after* the peak of the solar cycle. This may in part be due to a nonlinearity in the relation between AAVSO and Zurich numbers. It may also be related to the fact that the sunspot "count" is defined as a combination of the number of sunpots f and the number of sunpot groups g, according to

$$c = 10g + f \tag{15}$$

It may be that there is a difference in the proportion of groups to spots between the ascending and descending branches of the sunspot cycle, and a difference between an AAVSO observer's group count and the Zurich group count. This effect deserves further study.

References:

Schaefer, B. E. 1997, *J. Amer. Assoc. Var. Star Obs.*, **26**, 40.
Shapley, A. H. 1949, *Pub. Astron. Soc. Pacific*, **61**, 358.

Appendix: Derivation of equation (12)

First of all, (9) can be written

$$\tilde{k}_j = k_j \Big[\prod_{a=1}^{N} J^{-1} \sum_{p=1}^{J} \frac{1+\epsilon_{pa}}{1+\epsilon_{ja}}\Big]^{1/N} = k_j \prod_{a=1}^{N} \Big[J^{-1} \sum_{p=1}^{J} \frac{1+\epsilon_{pa}}{1+\epsilon_{ja}}\Big]^{1/N}$$

$$= k_j \prod_{a=1}^{N} \Big[1 + J^{-1} \sum_{p=1}^{J} \frac{\epsilon_{pa}-\epsilon_{ja}}{1+\epsilon_{ja}}\Big]^{1/N} \tag{16}$$

To compute the expected value of this, we note that any two errors ϵ_{ja} and ϵ_{pb} are independent for $a \neq b$, so the expected value of the $\prod_a$ is equal to the $\prod_a$ of the expected value, i.e.,

$$\langle \tilde{k}_j \rangle = k_j \prod_{a=1}^{N} \langle \Big[1 + J^{-1} \sum_{p=1}^{J} \frac{\epsilon_{pa}-\epsilon_{ja}}{1+\epsilon_{ja}}\Big]^{1/N} \rangle \tag{17}$$

Now I introduce the series expansion, for $q < 1$,

$$[1+q]^{1/N} = 1 + (1/N)q - \frac{N-1}{2N^2}q^2 + \frac{(N-1)(2N-1)}{6N^2}q^3 - \ldots \tag{18}$$

Using this, (16) becomes

$$\langle \tilde{k}_j \rangle \approx k_j \prod_{a=1}^{N} \langle \Big[1 + N^{-1} J^{-1} \sum_{p=1}^{J} \frac{\epsilon_{pa} - \epsilon_{ja}}{1 + \epsilon_{ja}}$$

$$- \frac{1}{2}(N-1) N^{-2} [J^{-1} \sum_{p=1}^{J} \frac{\epsilon_{pa} - \epsilon_{ja}}{1+\epsilon_{ja}}]^2 + \ldots \Big] \rangle$$

$$\approx k_j \prod_{a=1}^{N} \Big[1 + N^{-1} J^{-1} \sum_{p=1}^{J} \langle \frac{\epsilon_{pa} - \epsilon_{ja}}{1 + \epsilon_{ja}} \rangle \Big]. \quad (19)$$

For $p = j$, the term in the sum is of course zero. For $p \neq j$, we have ϵ_{pa} independent of ϵ_{ja}, so

$$\langle \frac{\epsilon_{pa}}{1 + \epsilon_{ja}} \rangle = \langle \epsilon_{pa} \rangle \langle \frac{1}{1 + \epsilon_{ja}} \rangle = 0 \times \langle \frac{1}{1 + \epsilon_{ja}} \rangle = 0. \quad (20)$$

Therefore the sum may be written

$$\sum_{p=1}^{J} \langle \frac{\epsilon_{pa} - \epsilon_{ja}}{1 + \epsilon_{ja}} \rangle = \sum_{p \neq j} \langle \frac{-\epsilon_{ja}}{1 + \epsilon_{ja}} \rangle = (J-1) \langle \frac{-\epsilon_{ja}}{1 + \epsilon_{ja}} \rangle \quad (21)$$

and (19) becomes

$$\langle \tilde{k}_j \rangle \approx k_j \prod_{a=1}^{N} \Big[1 - N^{-1} (J-1) J^{-1} \langle \frac{\epsilon_{ja}}{1 + \epsilon_{ja}} \rangle \Big]. \quad (22)$$

Now we can use the series, for $\epsilon < 1$,

$$\frac{1}{1+\epsilon} = 1 - \epsilon + \epsilon^2 - \epsilon^3 + \ldots, \quad (23)$$

to compute

$$\langle \frac{\epsilon_{ja}}{1 + \epsilon_{ja}} \rangle = \langle \epsilon_{ja} [1 - \epsilon_{ja} + \epsilon_{ja}^2 - \ldots] \rangle = \langle [\epsilon_{ja} - \epsilon_{ja}^2 + \epsilon_{ja}^3 - \ldots] \rangle$$

$$= 0 - \sigma_j^2 + \ldots \approx -\sigma_j^2. \quad (24)$$

For a reasonably large number of observers J and data N, and for $\sigma_j^2 << 1$, we have

$$\langle \tilde{k}_j \rangle \approx k_j \prod_{a=1}^{N} \Big[1 + \frac{1}{N} \frac{J-1}{J} \sigma_j^2 \Big] = k_j \Big[1 + \frac{1}{N} \frac{J-1}{J} \sigma_j^2 \Big]^N$$

$$\approx k_j \Big[1 + \frac{J-1}{J} \sigma_j^2 \Big] \approx k_j \Big[1 + \sigma_j^2 \Big]. \quad (25)$$

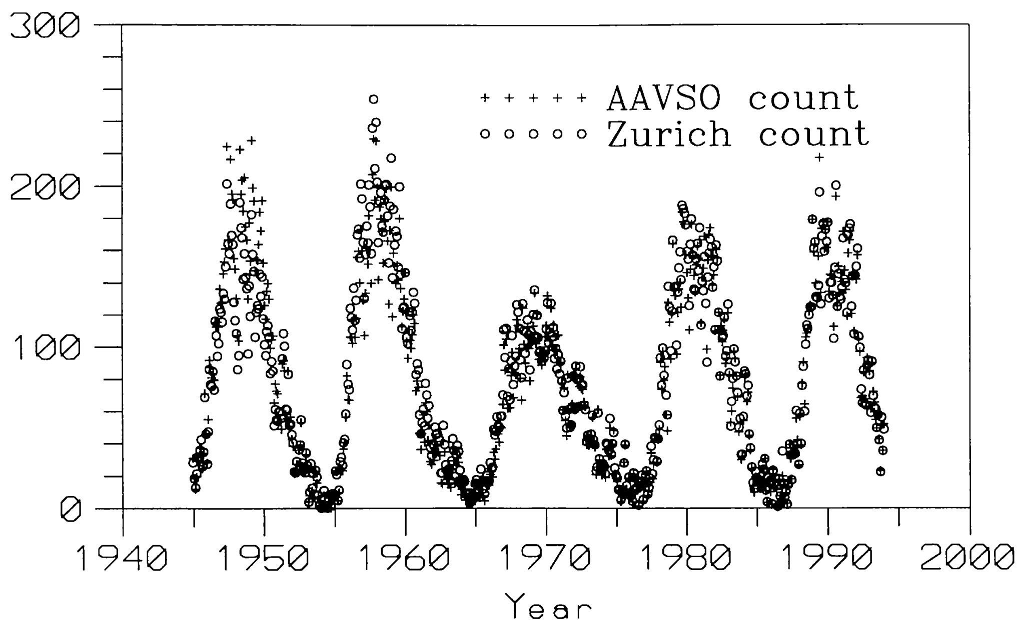

Figure 1. AAVSO sunspot counts (plus signs) compared to Zurich sunspot counts (circles).

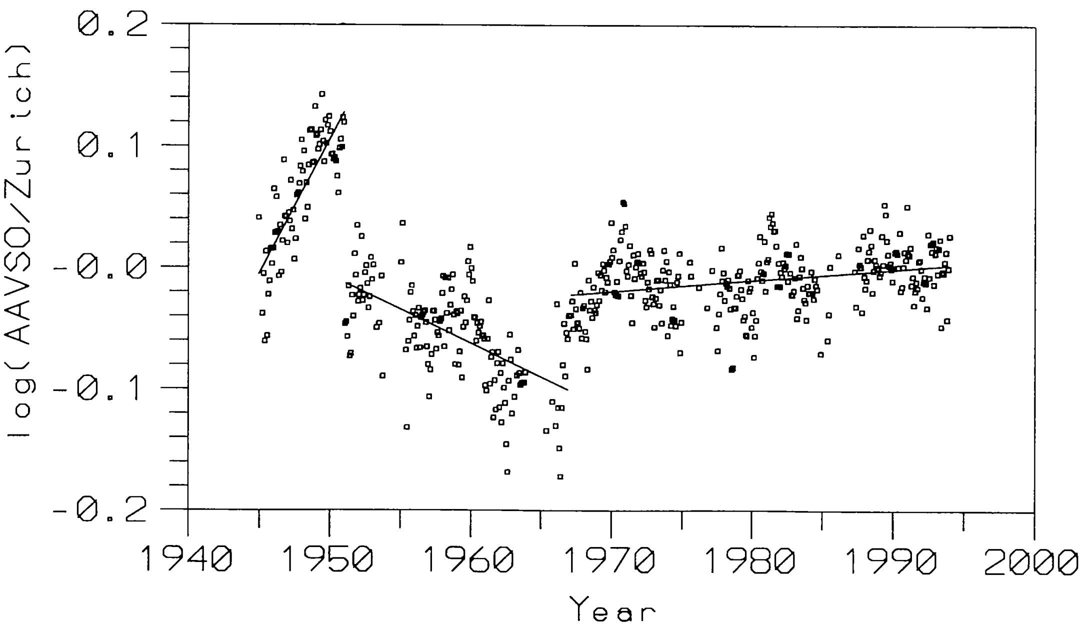

Figure 2. Logarithm of the ratio of AAVSO to Zurich sunspot numbers, $\log(R_A/R_Z)$, for the last 50 years. The straight lines are linear regression fits, showing the dramatic increase until 1951, an apparent decrease from 1951 to 1967, and a slow increase from 1967 to 1995.

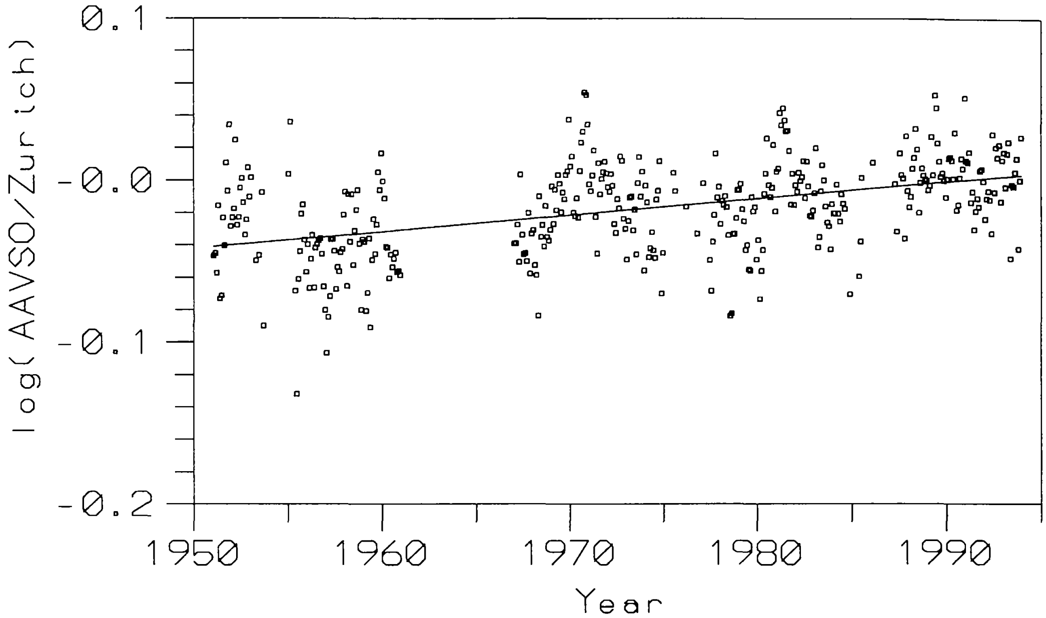

Figure 3. Same as Figure 2, but closeup view of the time span from 1950 to 1995, with data from 1963 to 1967 omitted.