AN AMPLITUDE ANALYSIS OF TU CASSIOPEIAE

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Abstract

TU Cas is a double-mode Cepheid variable star. The amplitudes of the two modes as observed from 1987 to 1990 are compared with previous data. A previously reported decrease in amplitude is not confirmed. This result favors stable mixed-mode pulsation over mode-switching.

1. Introduction

The double-mode Cepheid TU Cas is a variable star which exhibits two distinct periods in its light curve. These periods have been identified as the fundamental and the first overtone (Oosterhoff 1957). It had been suspected that a second overtone existed (Faulkner 1977), but this speculation has been discredited (Hodson *et al.* 1979).

The purpose of this paper is to discuss the possibility of a steady change in the amplitudes of the two modes. There has been suspicion that the ratio of these two amplitudes has decreased over the years (Hodson *et al.* 1979). This type of behavior would support one of the theories about the cause of the double-mode Cepheid pulsation, that theory being that the dominant periods of the star, fundamental and first overtone, tend to switch from one to the other during evolution toward the red (fundamental) or blue (overtone) on the H-R diagram, with double-mode behavior at the transition line between them (Stellingwerf 1975; Hodson and Cox 1976).

Hodson *et al.* (1979) studied the amplitude change of TU Cas over 67 years to test this theory. They pointed out that a lack of amplitude change over this time period would tend to support a stable, mixed-mode Cepheid. Their evidence indicated that mode-switching may be occurring.

2. Observations

The observations used in the analysis here comprise most of the data used by Hodson *et al.* (1979) and the more recent data (Mader *et al.* 1990) accumulated for the Maria Mitchell Observatory by the automatic photoelectric telescope (APT), Phoenix 10, at the Fairborn Observatory in Arizona (Boyd *et al.* 1986). Three time periods are analyzed separately: 1946-1959, 1962-1978, and 1987-1990. The data used by Hodson *et al.* from the 1911-1912 period are not used here due to the severe difficulties they had in calibrating the magnitude system. Differences of up to 0.3 magnitude exist between their different calibrations of the visual data.

The 1946-1959 data set consists of 28 measurements by Gordon and Kron (1947), 290 measurements from Worley and Eggen (1957), 44 measurements by Oosterhoff (1960), and 26 measurements by Weaver *et al.* (1960). The 1962-1978 data set consists of 6 measurements by Williams (1966), 14 measurements by Kwee and Braun (1967), 31 measurements by Takase (1969), and 60 unpublished measurements by Schmidt made from 1976 through 1978 (Stellingwerf 1990). The final data set, 1987-1990, is composed of 189 APT measurements. All of these measurements are in the V wavelength band.

In Oosterhoff's data, an anomalous point was discovered at JD 2436783.835 and was removed from the data set. This point differed by 0.2 magnitude from other points at that phase. Also, the 1987-1990 APT data contains some points which may be uncertain due to poor observing conditions at the time these observations were made. These data points have been indicated with an open circle in the figures and only half weight was given to these points in the following fitting process.

3. Method

The purpose of studying this older material (1946-1978) for TU Cas is to apply the same procedure used for reduction of the newer material (1987-1990) and to see if the results agree with those published by Hodson *et al.* (1979).

This method (Mader *et al.* 1990) begins by using Ferraz-Mello's (1981) Date Compensated Discrete Fourier Transform (DCDFT) to determine the strongest sinusoids in the data set. The same sinusoids were found to be prominent in the older and newer data. There are 15 frequencies, consisting of the fundamental with 3 higher harmonics, first overtone with one higher harmonic, and 9 cross-coupling terms.

The parameters of these sinusoids were found from a least squares solution which fits the magnitudes to a Fourier series (see Faulkner 1977, equation 1). I differ with Faulkner by using magnitudes rather than luminosities. The solution generates a constant term and the semi-amplitudes of 15 sine and 15 cosine terms. The least squares algorithm used in this analysis is based on Press *et al.* (1986, page 515). The period for the fundamental and first overtone were adopted from the *General Catalogue of Variable Stars* (GCVS)(Kholopov *et al.* 1985) as 2.139298 days for P_0 and 1.518300 days for P_1. The periods for higher harmonics and cross-coupling terms were calculated from these values.

4. Results

The subsets of the three data sets were found to have differing mean magnitudes. Hodson *et al.* (1979) also came upon this problem. To adjust for this, the least squares solution was found for each individual subset of data. The magnitudes in that subset were then modified to oscillate around V = 7.77 magnitude, the constant term in the 1987-1990 data. Table 1 shows the magnitude adjustment applied to each data subset.

Table 1. Magnitude Adjustments

	Year	*# of Observations*	*Correction to V*
1946-1959			
Gordon and Kron	1946	28	-0.13
Worley and Eggen	1952-1955	180	+0.09
Oosterhoff	1959	44	+0.02
Weaver *et al.*	1959	26	+0.01
1962-1978			
Williams	1962	6	-0.03
Kwee and Braun	1963	14	+0.04
Takase	1965-1966	31	+0.02
Schmidt	1976-1978	60	-0.01
1987-1990			
Mader, Arndt, and Hunt	1987-1990	189	0.0

To produce the fundamental light curve, all sinusoids are filtered from the data set except the fundamental and its harmonics. The filtered data set is then folded at the fundamental. Figures 1a, 1b, and 1c show this curve for each of the three data sets. The curved lines are the sum of the fundamental and its first three higher harmonics. The light curve of the first overtone is produced in a similar fashion wherein all sinusoids except the first overtone and its harmonic are filtered from the data set, which is then folded at the first overtone. Figures 2a, 2b, and 2c show the first overtone light curve for each of the three data sets used. The curved lines are the sum of the first overtone and its first higher harmonic. As mentioned previously, the open circles in Figures 1c and 2c of the APT data represent the uncertain points which have been half-weighted in the fitting process.

From these curved lines that describe the light curves, the amplitudes of the two periods, fundamental and first overtone, can be determined. These amplitudes are the differences between magnitudes at minimum and at maximum light, not the Fourier semi-amplitudes previously discussed. Table 2 shows the results of this analysis. Columns 1, 3, and 5 show our results as compared with those published by Hodson *et al.* (1979) which are shown in columns 2, 4, and 6. Columns 1 and 2 give the amplitudes of the fundamental at each different time period. Columns 3 and 4 list the amplitudes of the first overtone (A_0). In the last two columns, the ratios of the amplitudes of the first overtone to the fundamental are listed.

Table 2. Amplitudes and Amplitude Ratios for Fundamental and First Overtone Frequencies

Time Period	A_0		A_1		A_1/A_0	
	(1)	(2)	(3)	(4)	(5)	(6)
1946-1959	0.699	0.69	0.250	0.23	0.358	0.34
1962-1978	0.666	0.67	0.234	0.22	0.351	0.33
1987-1990	0.646		0.232		0.359	

(1), (3), (5) - this study
(2), (4), (6) - Hodson *et al.* 1979

Using error propagation techniques, the amount of error in determining the amplitudes of the light curves is calculated. For the earliest data set (1946-1959), the mean errors are ± 0.008 for P_0 and ± 0.006 for P_1. For the 1962-1978 data these errors are ± 0.010 for P_0 and ± 0.009 for P_1. The 1987-1990 data set has errors of ± 0.007 for P_0 and ± 0.006 for P_1. These errors set a limit to the accuracy of our amplitude ratio analysis.

5. Discussion and Conclusions

These recent results show that the amplitude ratio does not seem to display the suspected steady decrease. Comparing these results with those from Hodson, Stellingwerf, and Cox (1979) shows some agreement between the values, particularly those of the amplitudes of the fundamental. The individual amplitudes of the first overtone and fundamental do show decreases through time but the ratio does not.

We are studying a time period of 46 years. A decrease in the ratio of the two modes' amplitudes would suggest mode-switching occurring over this period. According to this method and analysis, TU Cas is not displaying mode switching. This evidence seems to favor stable mixed-mode pulsation.

Theoretical speculation about the classification of beat Cepheids has produced two hypotheses. It has been suggested (King *et al.* 1975) that beat Cepheids may be a class of stars separate from normal short-period Cepheids. This idea is based on the small masses and radii of beat Cepheids obtained from the analysis of period ratios (Stobie 1977). Also in support of this suggestion, TU Cas has been found to have a high gravity (Schmidt 1974) in comparison with normal Cepheids.

Other evidence seems to suggest that beat Cepheids may be a class of normal Cepheids displaying double-mode behavior as part of the evolutionary process. In support of this theory, the beat Cepheid U TrA has been found to have a temperature and gravity similar to other normal short-period Cepheids (Rodgers and Gingold 1973). In addition, Niva and Schmidt (1979) have shown that TU Cas has a radius comparable to the radius of the normal Cepheid SU Cas.

The evidence from this study seems to support the suggestion that beat Cepheids constitute their own class, separate from normal short-period Cepheids. However, this analysis encompasses only 46 years of TU Cas' behavior. It is possible that we have not covered a long enough time span to detect mode-switching that may occur on a longer time scale. A better analysis of the 1911-1912 data, if possible, would greatly improve the test. Future study of TU Cas may also help to clarify this question.

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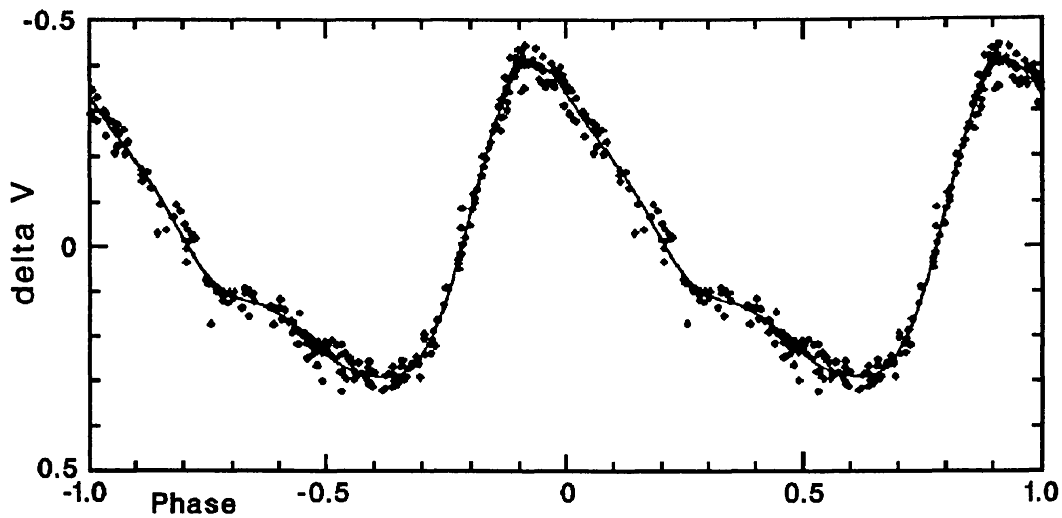

Figure 1a. The 1946-1959 data set for TU Cas folded at the fundamental. The first overtone, its harmonic, and all cross-coupling terms have been filtered. The curved line is the sum of the fundamental and its three higher harmonics.

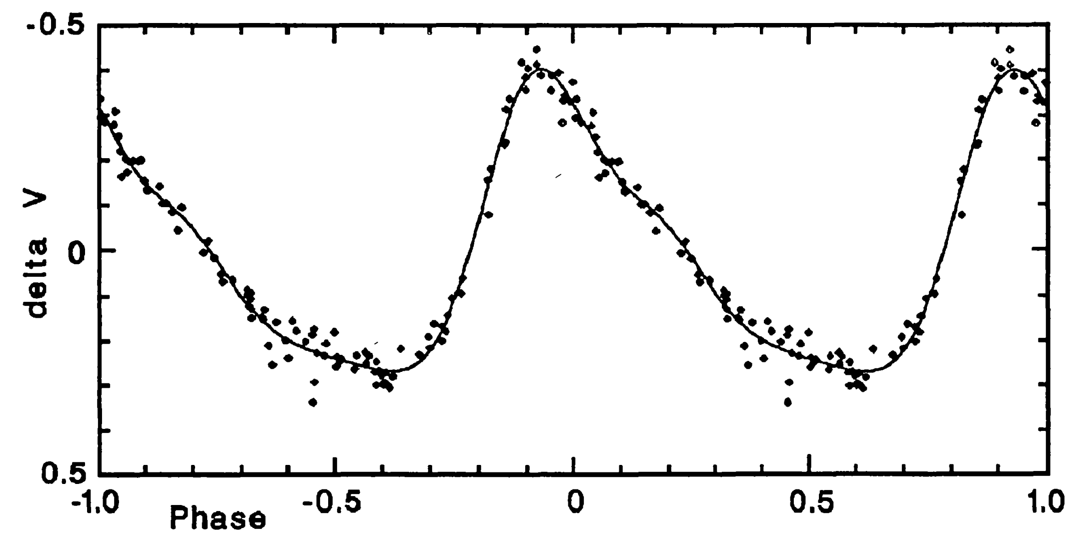

Figure 1b. The fundamental mode in the 1962-1978 data for TU Cas.

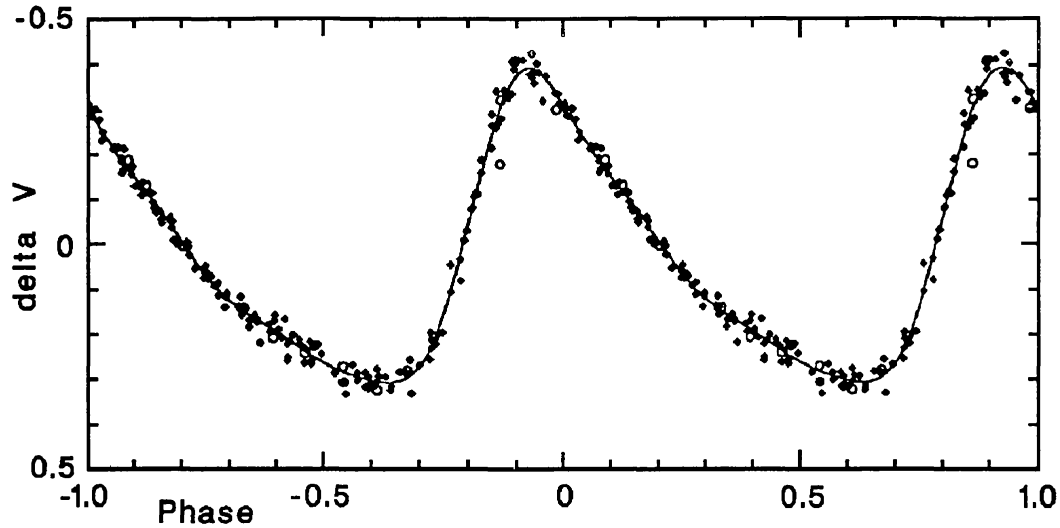

Figure 1c. The fundamental mode in the 1987-1990 data for TU Cas. Open circles indicate APT data which may be uncertain due to poor observing conditions at the time these observations were made. Half weight was given to these points in the fitting process.

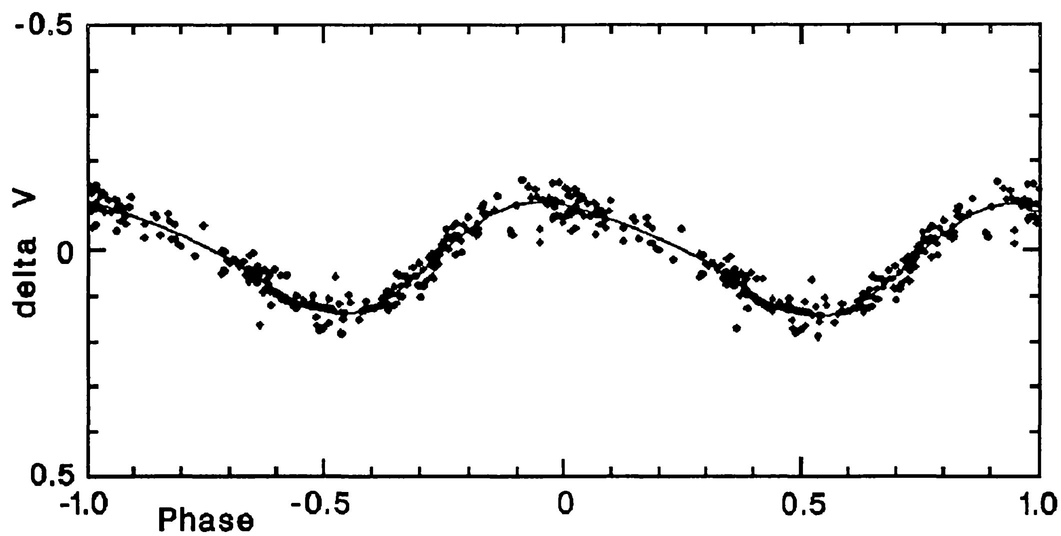

Figure 2a. The 1946-1959 data set for TU Cas folded at the first overtone. The fundmamental, its harmonics, and all cross-coupling terms have been filtered. The curved line is the sum of the first overtone and its three higher harmonics.

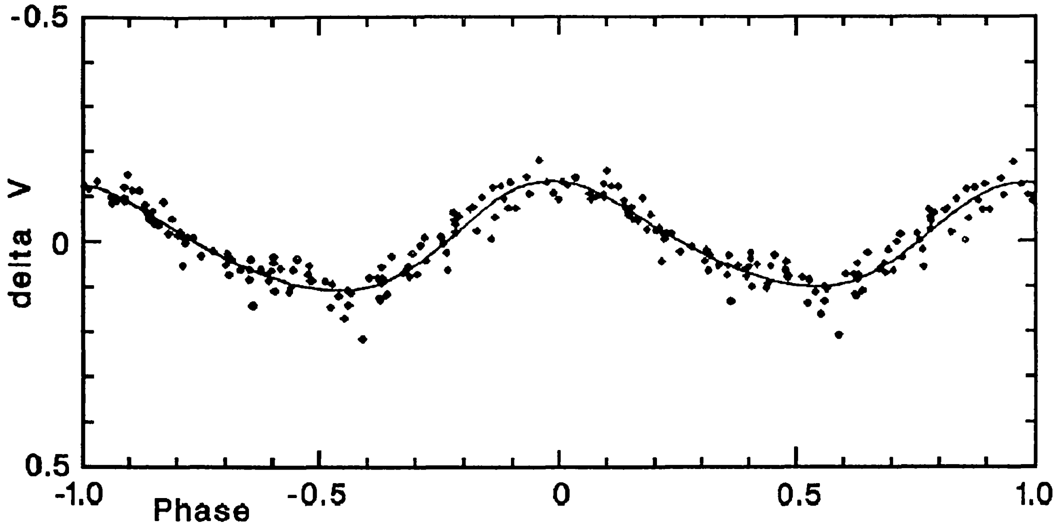

Figure 2b. The first overtone in the 1962-1978 data for TU Cas.

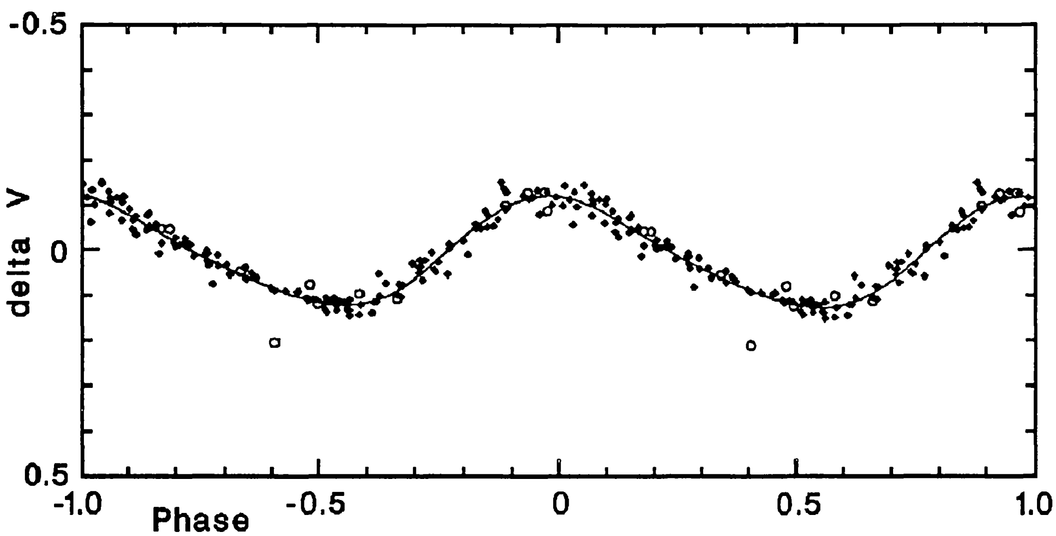

Figure 2c. The first overtone in the 1987-1990 data for TU Cas. Open circles indicate APT data which may be uncertain due to poor observing conditions at the time these observations were made. Half weight was given to these points in the fitting process.